MOISHEZON FOURFOLDS HOMEOMORPHIC
TO Q_C^4

Iku NAKAMURA

Introduction

In general, there are many different complex manifolds having the same underlying topological or differentiable structure. However there are a few exceptional cases where we can expect that homeomorphy to a given compact complex manifold implies analytic isomorphism to it, for instance, an irreducible compact Hermitian symmetric space. Among irreducible Hermitian symmetric spaces, the complex projective space P_C^n and a smooth hyperquadric Q_C^n in P_C^{n+1} seem to be nice exceptions which we can handle with algebraic methods. In [15] we studied the complex projective space P_C^n, while in the present article we study a smooth hyperquadric Q_C^n in P_C^{n+1} in the same way as in [15]. A goal we have in mind is the following

Conjecture MQ_n. *Any Moishezon complex manifold homeomorphic to Q_C^n is isomorphic to Q_C^n.*

The conjecture has been solved partially by Brieskorn [3] under the assumption that the manifold in question is *Kählerian* and odd-dimensional. In the even-dimensional *Kählerian* case, there still remains a possibility of manifolds of general type. Recently Kollár [7] and the author [13] solved Conjecture MQ_3 in the affirmative, each supplementing the other. Peternell [16][17] also asserts the same consequence. See [7,5.3.6].

Theorem 1. *Any Moishezon threefold homeomorphic to Q_C^3 is isomorphic to Q_C^3.*

The main purpose of the present article is to give a partial solution to the above conjecture MQ_4 in dimension 4. We prove,

Theorem 2. *Let X be a Moishezon fourfold homeomorphic to Q_C^4,*