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SINGULAR VARIATION OF DOMAIN AND SPECTRA OF THE LAPLACIAN WITH SMALL ROBIN CONDITIONAL BOUNDARY I.

Dedicated to Professor M.M. Schiffer on his 80th birthday

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1. Introduction

In this paper the author considers the following problem.

Let Ω be a bounded domain in $\boldsymbol{R}^2$ with smooth boundary $\partial\Omega$. Let $\tilde{w}$ be a fixed point in Ω. Let $B(\varepsilon, \tilde{w})$ be the ball of radius ε with the center $\tilde{w}$. We put $\Omega_\varepsilon=\Omega\backslash\overline{B(\varepsilon, \tilde{w})}$. Consider the following eigenvalue problem

$$(1.1)\qquad \begin{aligned} -\Delta u(x) &= \lambda u(x) && x\in\Omega_\varepsilon \\ u(x) &= 0 && x\in\partial\Omega \\ u(x)+k\,\varepsilon^\sigma \frac{\partial u}{\partial \nu_x}(x) &= 0 && x\in\partial B_\varepsilon\,. \end{aligned}$$

Here k denotes the positive constant. And σ is a non negative constant. Here $\frac{\partial}{\partial \nu_x}$ denotes the derivative along the exterior normal direction with respect to Ω_ε.

Let $\mu_j(\varepsilon)>0$ be the j-th eigenvalue of (1.1). Let μ_j be the j-th eigenvalue of the problem

$$(1.2)\qquad \begin{aligned} -\Delta u(x) &= \lambda u(x) && x\in\Omega \\ u(x) &= 0 && x\in\partial\Omega\,. \end{aligned}$$

Let $G(x,y)$ be the Green function of the Laplacian in Ω with the Dirichlet boundary condition on $\partial\Omega$ satisfying $-\Delta G(x,y)=\delta(x-y)$.

Main aim of this paper is to show the following Theorem 1. Let $\varphi_j(x)$ be the L^2 normalized eigenfunction associated with μ_j.

Theorem 1. *Fix $\sigma\in(0, 1)$. Fix j. Assume that μ_j is a simple eigenvalue. Then,*

$$(1.3)\qquad \begin{aligned} \mu_j(\varepsilon)-\mu_j &= 2\pi\, k^{-1}\, \varepsilon^{1-\sigma}\, \varphi_j(\tilde{w})^2 \\ &\quad +O(\varepsilon^{2-2\sigma}(\log \varepsilon)^2)\,. \end{aligned}$$