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MIXED TORELLI PROBLEM FOR TODOROV SURFACES

Dedicated to Professor H. Hironaka on the occasion of his sixtieth birthday

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Introduction

There is an approach to the Torelli problem by using degeneracy loci. Namikawa and Friedman succeeded to prove the generic Torelli theorem for curves [21] and the Torelli theorem for algebraic $K3$ surfaces [14] respectively in this direction.

In case of Todorov surfaces X, since the period map sending X to the Hodge structure on $H^2(X)$ has positive dimensional fibers ([29], [30], [31], [32], [33]) it is necessary to consider the mixed period map which sends X to the mixed Hodge structure on $H^2(X-C)$, where C is the unique canonical curve of X ([34], [23]). On the other hand, we can observe that Todorov surfaces are connected by "tame" degenerations and smooth deformations. It is the purpose of the present paper to try to solve mixed Torelli problem for Todorov surfaces by using the "tame" degenerations. At present we have formulated the problem inductively and obtained some results but we have not yet arrived at the final destination.

We give examples of "tame" degenerations of double covers of surfaces as Table 0 on the next page. Degenerations of type (I_1) in Table 0 are observed for Todorov surfaces and surfaces with $c_1^2=2p_g-3$, type (I_2) are observed for Kunev surfaces, and (II_1) are observed for surfaces on the Noether line ([36], [37]). Recently these phenomena are observed more widely ([18], [4], [2], [3]). So our present trial can be seen as a miniature of a more ambitious attempt, namely, to attack (mixed) Torelli problem for surfaces of general type via degeneracy loci.

§1 is a Hodge theoretic preliminary. We recall, after [28], the constructions of (filtered) cohomological mixed Hodge complexes whose hypercohomologies yield the terms in a mixed version of the Clemens-Schmid sequence. We distinguish filtrations corresponding to the openness of the varieties in question and to their singularity and see their relationships. We prove partial results on the exactness of the mixed Clemens-Schmid sequence.

§2 contains an observation that the moduli spaces of Todorov surfaces are