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ON A MIXED PROBLEM OF LINEAR ELASTODYNAMICS WITH A TIME-DEPENDENT DISCONTINUOUS BOUNDARY CONDITION

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1. Introduction

1.1. Problem and Main Result. Let Ω be a bounded domain in $\boldsymbol{R}^n$, $n\geqq 2$, with C^∞-boundary $\Gamma=\partial\Omega$. We consider the following mixed problem of linear elastodynamics:

Problem (D). *Find a vector function* $\boldsymbol{u}=(u^i(t, x))_{1\leqq i\leqq n}$ *satisfying*

$$\text{(D.1)} \qquad \left(\frac{\partial^2}{\partial t^2}+A(t)\right)\boldsymbol{u}=\boldsymbol{f} \quad \text{in} \quad \hat{\Omega}_T:=(0, T)\times\Omega\,, \ 0<T<\infty,$$

$$\text{(D.2)} \qquad \boldsymbol{u}(0, \cdot)=\boldsymbol{u}_0\,, \ \frac{\partial \boldsymbol{u}}{\partial t}(0, \cdot)=\boldsymbol{u}_1 \quad \text{in} \quad \Omega$$

with a time-dependent mixed boundary condition

$$\text{(D.3)} \qquad \begin{cases} \boldsymbol{u}=\boldsymbol{0} & \text{on} \quad \hat{\Gamma}_{D,T}:=\bigcup_{t\in[0,T]}\{t\}\times\Gamma_D(t), \\ B(t)\boldsymbol{u}=\boldsymbol{0} & \text{on} \quad \hat{\Gamma}_{N,T}:=\bigcup_{t\in[0,T]}\{t\}\times\Gamma_N(t) \end{cases}$$

for given $\boldsymbol{u}_0=(u_0^i(x))_{1\leqq i\leqq n}$, $\boldsymbol{u}_1=(u_1^i(x))_{1\leqq i\leqq n}$ *and* $\boldsymbol{f}=(f^i(t, x))_{1\leqq i\leqq n}$.

Here $A(t)$ and $B(t)$ are differential systems operating on $\boldsymbol{v}=(v^i(x))_{1\leqq i\leqq n}$ for each $t\in[0, T]$ defined by

$$\text{(1.1)} \qquad (A(t)\boldsymbol{v})^i=-\frac{\partial}{\partial x^j}\left(a^{ijkh}(t, x)\frac{\partial v^k}{\partial x^h}\right) \quad \text{in} \quad \Omega\,,$$

$$\text{(1.2)} \qquad (B(t)\boldsymbol{v})^i=\nu_j(\dot{x})a^{ijkh}(t, \dot{x})\frac{\partial v^k}{\partial x^h} \qquad \text{on } \Gamma \quad \text{for } 1\leqq i\leqq n$$

where $a^{ijkh}(t, x)$ are real-valued C^∞-functions on $\overline{\hat{\Omega}}_T$ with *symmetry relations*

$$\text{(1.3)} \qquad a^{ijkh}(t, x)=a^{khij}(t, x) \qquad \text{for} \quad 1\leqq i, j, k, h\leqq n\,,$$

and $\boldsymbol{\nu}(\dot{x})=(\nu_j(\dot{x}))_{1\leqq j\leqq n}$ denotes the unit outer normal to Γ at $\dot{x}\in\Gamma$. (Super- and