# ON SOME ABSTRACT DEGENERATE PROBLEMS <br> OF PARABOLIC TYPE-3: APPLICATIONS TO LINEAR AND NONLINEAR PROBLEMS 

Dedicated to Professor B. Pini on his 70th birthday

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## 0. Introduction

In papers [11], [12] (which will be called henceforth part I and part II, respectively) we developed some abstract results about existence and uniqueness of solutions of Cauchy problems related to degenerate evolution equations which might be put under the general pattern

$$
\begin{equation*}
\frac{d}{d t}(M(t) u(t))+L(t) u(t)=f(t, u(t)), \text { for every } t \in[0, \tau] \tag{0.1}
\end{equation*}
$$

where $L, M$ are linear operators, possibly depending upon $t$ (we shall call it the time variable, as opposed to the variable $x$ of the functional space in which $u(t)$ lies, to be referred to as the space variable); more precise assumptions will be made in each particular case.

In parts I and II the technique involved deeper abstraction, leading to study of ( 0.1 ) as a particular case of more general, quite algebraically-looking equations:

$$
\begin{equation*}
B M u+L u=F(u) \tag{0.2}
\end{equation*}
$$

or even

$$
\begin{equation*}
B M u=f(u) \tag{0.3}
\end{equation*}
$$

here, we recall or restate in the specific case those results: while doing so, we hope to make this part reasonably self-contained.

A remark about style: the assumptions which work are sometimes rather lengthy, the details cumbersome; we choose therefore not to seek maximal generality; in some applications, we do not emphasize degeneration, even though we might introduce it throughout, and stress nonlinearity. Sometimes assumptions could be weakened: we often label 'regular' ( $=$ 'of cass $C^{(1)}$ ') something which could have been differentiable, plus something better, or even less, as well. At first, problems and equations are formally stated, i.e. without precise assumptions

