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CONJUGACY CLASSES AND REPRESENTATION GROUPS

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Let D be a conjugacy class of a finite group G, and H a (finite) central extension of G. In the first section of the paper, we investigate how D is extended in H. Let ϕ be the homomorphism of H to G. Then, there exist a group H_0 and epimorphisms $\phi_1: H\to H_0$ and $\phi_2: H_0\to G$ such that $\phi=\phi_1\phi_2$, that $\phi_2^{-1}(D)=E_1\cup\cdots\cup E_n$ with conjugacy classes E_i of H_0 with $|E_i|=|D|$ (i.e., D splits completely in H_0), and that $\phi_1^{-1}(E_i)=C_i$ with a conjugacy class C_i of H with $|C_i|=e|D|$ for every i. e is called the (covering) multiplicity of D in H. Especially, when H is a representation group of G, we can show that e is equal to $|M|/|M_0|$ where M is the Schur multiplier of G and M_0 is a subgroup of M consisting of all cohomology classes that split over D. In the second section of the paper, we investigate the structure of a group or of a conjugacy class of a group with respect to inner automorphisms. An algebraic system which is the abstraction of a group with inner automorphisms as operations is called a p.s. set (or, a pseudosymmetric set). We show that all representation groups of G are isomorphic with respect to inner automorphisms, i.e., as p.s. sets, that every conjugacy class of a central extension of G is a homomorphic image of a conjugacy class of a representation group of G, and that the multiplicity e given in the above divides the order of the Schur multiplier of G. As an application, we obtain a criterion for a p.s. set to be a conjugacy class of a group, using which we can find a class of exceptional transitive p.s. sets of orders $n(n-1)(n-2)/2$, $(n\geq 5)$.

1. Central extensions of conjugacy classes

Proposition 1. *Let H be a group, Z a subgroup of H contained in the center of H, and C a conjugacy class of H. Let $Z_0=\{z\in Z\mid zC=C\}$. Then, $Z_0=[a, H]\cap Z$ for any element a in ZC. If $\{z_i\}$ is a representative system of Z/Z_0, then ZC is a union of conjugacy classes C_i where $C_i=z_iC$ and $C_i\neq C_j$ if $i\neq j$. Thus, ZC is a union of conjugacy classes of the same order.*

Proof. In the following, we denote $y^{-1}xy$ by $x\circ y$. So, $C=c\circ H$ with c in C. An element z of Z belongs to Z_0 if and only if $zc=c\circ x$ for some element x in H, which implies that $z=[c, x]$. Hence, $Z_0=[c, H]\cap Z$. On the other