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HOMEOMORPHISMS WITH MARKOV PARTITIONS

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1. Introduction

The Markov partition in dynamical systems suplies us important informations (for examples, for studies of equilibrium states [5] and zeta functions [24]).

Such a partition was first constructed for Anosov diffeomorphisms by Ja.G. Sinai [35]. After that, R. Bowen [5] showed the existence of Markov partition on nonwandering sets of Axiom A diffeomorphisms. In these papers the notion of canonical coordinate play an important role to construct Markov partitions. K. Hiraide [20], in purely topological setting, proved the existence of Markov partition for expansive homeomorphisms with POTP by constructing canonical coordinates. For example, every expansive automorphism of a solenoidal group has POTP, and hence a cononical coordinate as well as a Markov partition (N. Aoki [2], [3] and [20]).

However homeomorphisms with Markov partitions do not necessarily have canonical coordinates. In fact, every pseudo-Anosov map has a Markov partition and does not have cononical coordinates (see paragraphs 9 and 10 of [17]).

Thus it is natural to ask what kind of expanisive homeomorphisms have Markov partitions. The purpose of this paper is to give necessary and sufficient conditions for expansive homeomorphisms to have Markov partitions. More precisely we can describe our result as follows;

Theorem. *Let X be a compact metric space and f be an expansive homeomorphism of X with expansive constant c^*. Then the following conditions are equivalent;*

(I) *there exists $c>0$ with $2c\leqq c^*$ such that for every $x\in X$ there exists an $\eta=\eta(x)>0$ such that $\{Y_c(y)\cap B_\eta(x)\mid y\in B_\eta(x)\}$ is finite,*

(II) *there exists $c<0$ with $2c\leq c^*$ such that for every $x\in X$ there exists a $\delta=\delta(x)>0$ such that $\{Z_c(y)\cap B_\delta(x)\mid y\in B_\delta(x)\}$ is finite,*

(III) *(X, f) has SPOTP,*

(IV) *(X, f) has a Markov partition.*

The proof will be give in section 3 and the auxiliary results used in the proof will be prepared in section 2. We shall describe in section 4 some applications