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RIGHT PERFECT RINGS WITH THE EXTENDING PROPERTY ON FINITELY GENERATED FREE MODULES

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In [3], [4] Harada has studied the following conditions:

(*) Every non-small right R-module contains a non-zero injective submodule.

(*)* Every non-cosmall right R-module contains a non-zero projective direct summand.

And he has found two new classes of rings which are characterized by ideal theoretic conditions: one is perfect rings with (*) and the other one is semi-perfect rings with (*)*. In [9], Oshiro has studied these rings by using the lifting and extending property of modules, and defined H-rings and co-H-rings related to (*) and (*)*, respectively.

A ring R is called a right H-ring if R is right artinian and R satisfies (*). Dually, R is called a right co-H-ring if R satisfies (*)* and the ACC on right annihilator ideals.

A right R-module M is said to be an extending module if for any submodule A of M there is a direct summand A^* of M containing A such that A_R is essential in A^*_R. If this "extending property" holds only for uniform submodules of M, so M is called a module with the extending property for uniform modules.

The following theorem is proved by Oshiro in [9, Theorem 3.18].

Theorem. *For a ring R the following conditions are equivalent:*

1) *R is a right co-H-rings.*

2) *Every projective right R-module is an extending module.*

3) *Every right R-module is expressed as a direct sum of a projective module and a singular module.*

4) *The family of all projective right R-modules is closed under taking essential extensions, i.e. for any exact sequence* $O \to P \xrightarrow{\varphi} M$, *where P is projective and* im φ *is essential in M, M is projective.*

In this paper we shall consider the case that R is a right perfect ring with