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e-INVARIANTS ON THE STABLE COHOMOTOPY GROUPS OF LIE GROUPS

Dedicated to Professor Masahiro Sugawara on his 60th birthday

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0. Introduction

The e-invariant was introduced by Adams [2] and Toda [13]. We know that this invariant is very useful and powerful in homotopy theory.

In this paper we will calculate e-invariants of certain elements of stable cohomotopy groups of Lie groups, more precisely, elements which arise from the Hopf construction of representations of Lie groups. We use the definition of the (complex) e-invariant in terms of the Chern character and resultly which is expressed as a tuple of rationals. These will give us informations on orders of above elements. Actually we will observe that our invariants behave well among Lie groups of low rank.

Our method depends on classical Theorems of Adams [1] and the result of a homotopy type of a Thom complex by Held-Sjerve [5]. To compute e-invariants of Lie groups of low rank, we will utilize the determination of the image of Chern character by T. Watanabe [15].

This paper is organized as follows. In Section 1, we will define an e-invariant on the stable cohomotopy group of a Lie group. In Section 2 we will introduce a theorem by which we can obtain our result on the Hopf-construction of a representation. In Section 3, we will show how to compute e-invariants concretely by computing them of few examples. In Section 4, we will consider simple applications of previous sections.

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1. A definition of an e-invariant

Let G be a compact connected Lie group with torsion-free fundamental group. Assume that we are given an element μ of the 0-th reduced stable cohomotopy group $\tilde{\pi}^0(G)$. We shall define an e-invariant of μ in terms of the Chern character. Hodgkin's Theorem [6] states that $K^*(G)$ has no torsion. This implies that μ induces a trivial homomorphism on the K-cohomology and the rational cohomology since $\tilde{\pi}^0(G)$ is finite. Thus we obtain the following