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A STOCHASTIC RESOLUTION OF A COMPLEX MONGE-AMPÈRE EQUATION ON A NEGATIVELY CURVED KÄHLER MANIFOLD

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1. Introduction

The Dirichlet problem for the complex Monge-Ampère equation on a strongly pseudo-convex domain of $\boldsymbol{C}^n$ was studied and solved by Bedford-Taylor [3]. The same problem for the Monge-Ampère equation on a negatively curved Kähler manifold has been recently proposed and solved by T. Asaba [2]. The main purpose of this paper is to solve the equation by using a method of the stochastic control presented by B. Gaveau [6].

Let M be an n-dimensional simply connected Kähler manifold with metric g whose sectional curvature K satisfies

$$-b^2 \leqq K \leqq -a^2$$

on M for some positive constants b and a. ω_0 denotes the associated Kahler form. We denote by $M(\infty)$ the Eberlein-O'Neill's ideal boundary of M and we always consider the cone topology on $\bar{M}=M \cup M(\infty)$ (see [4] for these notions). T. Asaba formulated the Monge-Ampère equation on M in the following manner:

We write PSH(D) for the family of locally bounded plurisubharmonic functions defined on a complex manifold D. When $u \in \mathrm{PSH}(D)$, the current $\underbrace{(dd^c u)^n = dd^c u \wedge \cdots \wedge dd^c u}_{n\text{-copies}}$ of type-(n, n) is defined as a positive Radon measure on D. Therefore, for given functions $f \in C(M)$ and $\varphi \in C(M(\infty))$, the complex Monge-Ampère equation

$$(1) \qquad \begin{cases} u \in \mathrm{PSH}(M) \cap C(\bar{M}) \\ (dd^c u)^n = f \omega_0^n / n! \qquad \text{on } M \\ u|_{M(\infty)} = \varphi \end{cases}$$

can be considered. T. Asaba found a unique solution of (1) by imposing the following condition on f: there exist two positive constants μ_0 and C_0 such that