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Osaka J. Math.
21 (1984), 91–111

AN APPROXIMATION THEOREM FOR MARKOV PROCESSES

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(Received April 5, 1982)

1. Introduction

In [1], Watanabe proved that for every Markov process X, under some conditions, there exists a sequence of regular step processes (R.S.P.) X^n such that the resolvents of X^n converge weakly to the resolvent of X. Under some supplementary conditions we shall prove that the distributions of X^n converge to the distribution of X. An intuitive description of X^n is as follows: X and X^n start frcm the same state x_0 (we mean that X_0 and X_0^n have the same distribution). If X remains closed to x_0 for a time T_n (that is, $d(x_0, X_t)<\frac{1}{n}$ for all $t<T_n$ and $X_{T_n}=x_1$, with $d(x_0, x_1)\geqq\frac{1}{n}$), then $X_t^n=x_0$ for all $t<D_n$, with D_n an exponentially distributed holding time with same mean value as T_n (T_n is generally not exponentially distributed). Then X^n jumps in x_1 (we mean that X_{T_n} and $X_{D_n}^n$ have the same distribution), and so on.

The rigorous construction of X^n and Watanabe's result are presented in the beginning of the paper. The theorem following this construction is the main result of the paper.

2. Main results

Let E be a locally compact with countable base space (L.C.C.B.), $\mathcal{U}$ an open base and d any metric of E. For each n we can choose the system U_i^n, $i\in N$ and V_i^n, $i\in N$ of sets in $\mathcal{U}$ satisfying the following conditions:

(1) Each $\bar{U}_i^n$ is compact and $d(U_i^n)<\frac{1}{n}$ $(d(A)=\sup(d(x,y);\, x, y\in A))$;

(2) $V_i^n\subseteq U_i^n$;

(3) $\bigcup_i V_i^n=E$;

(4) For every compact set K only a finite number of V_i^n intersect with K.

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, X_t, \theta_t, P^x)$ be a standard process with state space E and $(U_\alpha)_{\alpha>0}$ be the resolvent of X. We now define σ_k^n by