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ON SEMI-FIELD PLANES OF EVEN ORDER

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1. Introduction

Let π be a non-Desarguesian semi-field plane with an autotopism group G and let $u(\pi)$ denote the number of the orbits of G on the points not incident with any side of the autotopism triangles.

In their paper [9], M.J. Kallaher and R.A. Liebler have conjectured that $u(\pi)\geq 5$ and they have proved that the conjecture is true if G is solvable and the order of π is not 2^6.

In this paper we treat semi-field planes of even order whose autotopism groups are not necessarily solvable and prove the following.

Theorem 1. *Let π be a non-Desarguesian semi-field plane of order 2^r. If r is not divisible by 4, then $u(\pi)\geq 5$.*

The proof requires the use of the Kallaher-Liebler's theorem mentioned above and the following lemma which we prove in section 3.

Lemma 2. *Let π be a non-Desarguesian semi-field plane of order 2^6 with a solvable autotopism group. Then $u(\pi)\geq 5$.*

2. Notations and preliminaries

Our notation is largely standard and taken from [3] and [6]. Let G be a permutation group on Ω. For $X\leq G$ and $\Delta\subset\Omega$, we define $F(X)=\{\alpha\in\Omega\mid\alpha^x=\alpha \text{ for all } x\in X\}$, $X(\Delta)=\{x\in X\mid\Delta^x=\Delta\}$, $X_\Delta=\{x\in X\mid\alpha^x=\alpha \text{ for all } \alpha\in\Delta\}$ and $X^\Delta=X(\Delta)/X_\Delta$, the restriction of X on Δ. When X is a collineation group of a projective plane, we denote by $F(X)$ the set of fixed points and fixed lines of X.

Lemma 2.1. *Let G be a transitive permutation group on a finite set Ω, H a stabilizer of a point of Ω and M a nonempty subset of G. Then $|F(M)|=|N_G(M)|\times|ccl_G(M)\cap H|/|H|$. Here $ccl_G(M)\cap H=\{g^{-1}Mg\mid g^{-1}Mg\subset H, g\in G\}$.*

Proof. Set $W=\{(L,\alpha)\mid L\in ccl_G(M), \alpha\in F(L)\}$ and $W_\alpha=\{L\mid L\in ccl_G(M),$