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ON HAKEN'S THEOREM AND ITS EXTENSION

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

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1. Introduction

It is an interesting problem to investigate how a surface in a 3-manifold M intersects a fixed Heegaard surface in M. In this direction, the first basic work was done by W. Haken [1], and later W. Jaco [2] proved Haken's theorem in [1] in a complete form by using a theory of hierarchy for planar surfaces. In contrast with their works, the main purpose of this paper is to discuss a relationship of 2-sided projective planes in a 3-manifold M and a fixed Heegaard surface in M. In our discussion, a certain property which planar surfaces and Möbius strips with holes have in common plays an important role and then an observation on such a property enables us to prove the following;

Main Theorem 1. *Let M be a closed connected 3-manifold with a fixed Heegaard splitting (M, F) of genus g. Then the following holds;*

(1) *If there exists a 2-sided projective plane P in M, then there exists a 2-sided projective plane P' in M such that $F \cap P'$ is a single circle. In particular, if M is irreducible, then P' is isotopic in M to P.*

(2) *If M contains an incompressible 2-sphere, then there exists an incompressible 2-sphere S^2 in M such that $F \cap S^2$ is a single circle which is not contractible in F.*

It will be noticed that the second assertion of the Main Theorem 1 is the one of Haken's theorem in [1], [2] and so the above theorem includes Haken's theorem and that recently the author proved in [3] that every closed connected 3-manifold, with a Heegaard splitting of genus 2, which admits a 2-sided embedding of the projective plane P^2, is homeomorphic to $P^2 \times S^1$.

Throughout this paper, spaces and maps will be considered in the piecewise-linear category, unless otherwise specified. S^n, D^n denote n-sphere, n-disk, respectively. Closure, interior, boundary are denoted by $\mathrm{cl}(\cdot)$, $\mathrm{int}(\cdot)$, $\partial(\cdot)$, respectively. If X, Y are spaces with $X \supset Y$, then $N(Y, X)$ denotes a regular neighborhood of Y in X.

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