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ON MODULES WITH LIFTING PROPERTIES

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We have studied the lifiting property on a direct sum of completely indecomposable and cyclic hollow modules over a ring R in [8].

In this note, we shall define the lifting property of decompositions with finite direct summands in §1 and give characterizations of this in terms of endomorphism rings of R-modules in §2. We shall study, in §3, R-modules with lifting properties and show that they are very closed to R-modules with direct decomposition of completely indecomposable and cyclic hollow modules when R is right noetherian.

We shall give the dual results for the extending property and its applications in the forthcoming papers.

1. Definitions

Throughout this paper we assume that a ring R contains an identity and every R-module M is a unitary right R-module. We recall here definitions in [8].

If $\mathrm{End}_R(M)$ is a local ring, we call M a *completely indecomposable module*. We denote the Jacobson radical and an injective envelope of M by $J(M)$ and $E(M)$, respectively. By $\bar{M}$ we denote $M/J(M)$. If N is a submodule of M and $N/J(N)$ is canonically monomorphic into $M/J(M)$, then we mean $\bar{N}$ both $N/J(N)$ and the image of $N/J(N)$ into $M/J(M)$.

If $J(M)$ is a unique maximal and small submodule in M, we call M a *cyclic hollow module* (actually M is cyclic). If, for each simple submodule A of $\bar{M}$, there exists a completely indecomposable and cyclic hollow direct summand M_1 of M such that $\bar{M}_1=A$, then we say M has *the lifting property of simple modules* (*modulo radical*). More generally, if for any direct summand B of $\bar{M}$, there exists a direct summand M' of M such that $\bar{M}'=B$, we say M has *the lifting property of direct summands* (*modulo radical*). Finally if, for any finite decomposition of $\bar{M}$; $\bar{M}=C_1\oplus C_2\oplus\cdots\oplus C_n$, there exists a decomposition of M; $M=M_1\oplus M_2\oplus\cdots\oplus M_n$ such that $\bar{M}_i=C_i$, we say M has *the lifting property of decompositions with finite direct summands* (*modulo radical*). If the above property is satisfied for any direct decompositions, we say M has *the lifting property of decompositions* (*modulo radical*).

We recall the definition in [8].