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ON THE FUNDAMENTAL SOLUTION FOR A DEGENERATE HYPERBOLIC SYSTEM

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Introduction. Let $S[m_1, m_2]$ denote the set of all C^∞-symbols $a(t, x, \xi)$ on $[0, T]\times R_x^n\times R_\xi^n$ $(0<T\leqq 1)$ such that

$$|D_t^j D_\xi^\alpha D_x^\beta a(t, x, \xi)|\leqq C_{j,\alpha,\beta}\langle\xi\rangle^{m_1-|\alpha|}(t+\langle\xi\rangle^{-\omega})^{m_2-j} \tag{0.1}$$

for constants $C_{j,\alpha,\beta}$, where $\langle\xi\rangle=(1+|\xi|^2)^{1/2}$ and $\omega=1/(l+1)$ with an integer $l>0$.

Consider a hyperbolic operator of first order:

$$\boldsymbol{L} = \boldsymbol{D}_t-t^l\begin{bmatrix}\mu_1(t, X, D_x) & & 0\\ & \ddots & \\ 0 & & \mu_m(t, X, D_x)\end{bmatrix}+\boldsymbol{B}(t)\,, \tag{0.2}$$

where μ_j, $j=1, \cdots, m$ are real valued and satisfy

$$\begin{cases}\text{i)} & \mu_j(t, x, \xi)\in S[1, 0]\\ \text{ii)} & |\mu_j(t, x, \xi)-\mu_k(t, x, \xi)|\geqq c\langle\xi\rangle \qquad (j\neq k)\end{cases} \tag{0.3}$$

for a constant $c>0$, and the symbol $\sigma(\boldsymbol{B}(t))(x, \xi)$ of the lower order operator $\boldsymbol{B}(t)$ satisfies

$$\sigma(\boldsymbol{B}(t))(x, \xi)\in S[0, -1]\,. \tag{0.4}$$

The purpose of the present paper is to construct the fundamental solution $\boldsymbol{E}(t, s)$ $(0\leqq s\leqq t\leqq T_0)$ of the Cauchy problem

$$\begin{cases}\boldsymbol{L}U = \Phi(t) & \text{on} \quad [s, T_0]\,,\\ U|_{t=s} = \Psi\end{cases} \tag{0.5}$$

for a small constant T_0 $(0<T_0\leqq T)$. It should be noted that the operator $\boldsymbol{L}$ is degenerate at $t=0$ and $\boldsymbol{B}(t)$ is not uniformly bounded on $[0, T]$ as a family of pseudo-differential operators with parameter $t\in[0, T]$.

To construct $\boldsymbol{E}(t, s)$, we find first the perfect diagonalizer $\boldsymbol{N}(t)$ such that the symbol $\sigma(\boldsymbol{N}(t))(x, \xi)$ belongs to $S[0, 0]$ and