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RIBBON KNOTS AND RIBBON DISKS

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For a ribbon knot, we will define, in §1, the ribbon disk pair associated with it. On the other hand, J.F.P. Hudson and D.W. Sumners gave a method to construct a disk pair [2], [13]. In §1 and 2, we will generalize their construction and show that a ribbon disk pair is obtained by our construction and vice versa.

In [10], C.D. Papakyriakopoulos proved that the complement of a classical knot is aspherical. As an analogy of this, we will prove, in §3, that the compelment of a ribbon disk is aspherical, and it follows from this fact that the fundamental group of a ribbon knot complement has no element of finite order. In the final section, we will calculate the higher homotopy groups of a higher-dimensional ribbon knot complement, and in Theorem 4.4 we show that *a ribbon n-knot for $n\geqq 3$ is unknotted if the fundamental group of the knot complement is the infinite cyclic group.* This result is proved independently by A. Kawauchi and T. Matumoto [5].

Throughout the paper, we work in the piecewise-linear category although the results remain valid in the smooth category.

1. Preliminaries

1.1. By S^n we denote an n-sphere, and by B^n or D^n an n-disk. By ∂M, int M and cl M we denote the boundary, the interior and the closure of a manifold M respectively. In this paper, every submanifold in a manifold is assumed to be locally flat. If $\partial M\neq\emptyset$, by $\mathcal{D}M$ we mean the *double* of M, i.e. $\mathcal{D}M$ is obtained from the disjoint union of two copies of M by identifying their boundaries via the identity map. For a subcomplex C in a manifold M, $N(C; M)$ is a regular neighbourhood of C in M. By a *pair* (M, W) we denote a manifold M and a *proper* submanifold W in M, i.e. $W\cap\partial M=\partial W$. An *n-disk pair* is a pair (M, W) such that M is a disk and W an n-disk. Two pairs (M_1, W_1) and (M_2, W_2) are *equivalent* if there exists a homeomorphism from M_1 to M_2 which maps W_1 to W_2, and we will identify two equivalent manifold pairs. Let $\mathcal{D}(M, W)=(\mathcal{D}M, \mathcal{D}W)$ and $\partial(M, W)=(\partial M, \partial W)$. We denote the unit interval [0, 1] by I, and the Eu-