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# SOME REMARKS ON THE EQUATION $y_{tt}-\sigma(y_x)y_{xx}-y_{xtx}=f$

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## 1. Introduction

In [4], Greenberg, MacCamy and Mizel considered the following initial-boundary value problem which we denote by (Pr.I):

$$y_{tt}-\sigma(y_x)y_{xx}-y_{xtx}=f, \qquad (x,t)\in(0,1)\times(0,\infty)\,, \tag{1.1}$$

$$y(0,t)=y(1,t)=0, \qquad t\in(0,\infty)\,, \tag{1.2}$$

$$y(x,0)=y_0(x),\ y_t(x,0)=y_1(x), \qquad x\in(0,1)\,, \tag{1.3}$$

where $y$ is an unknown function and $y_0$, $y_1$ and $f$ are given functions. (For the physical meaning of this problem, see [4].) They established the existence, uniqueness and stability of smooth solutions of (Pr.I) under the assumptions that $\sigma$ is a positive $C^2(-\infty, \infty)$ function and that initial data $y_0$ and $y_1$ are, respectively, $C^4[0,1]$ and $C^2[0,1]$ functions vanishing together with their second derivatives at zero and one. The method of proof used in [4] are rather complicate and heavily depends upon some special properties of the Green function of the heat equation. (See also Davis [1], Ebihara [2] and Greenberg [3].)

The main purpose of the present paper is to weaken the assumptions in [4] and give a simplified proof of the existence, uniqueness and stability of smooth solutions of (Pr.I). We assume that $\sigma$ is a non-negative $C^1(-\infty, \infty)$ function and that initial data $y_0$ and $y_1$ are, respectively, $C^2[0,1]$ and $C[0,1]$ functions such that $y_0(0)=y_0(1)=y_{0,xx}(0)=y_{0,xx}(1)=0$ and $y_1(0)=y_1(1)=0$. Under these assumptions, we choose a Banach space $X_0=\{y\in C[0,1];\ y(0)=y(1)=0\}$ and regard $y$ as a map from $[0,\infty)$ to $X_0$. Let $A=\partial^2/\partial x^2$. We can formally rewrite (Pr.I) in an abstract form:

$$\begin{cases} y_{tt}-Ay_t-By=f, & t\in(0,\infty)\,, \\ y(0)=y_0,\ y_t(0)=y_1\,, \end{cases} \tag{1.4}$$

where $B$ is a nonlinear operator defined by $By(x)=\sigma(y_x(x))y_{xx}(x)$. Set $u=y_t$ and $v=Ay$. Then (1.4) is equivalent to the following: