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ON THE GROWTH OF SOLUTIONS OF SEMI-LINEAR DIFFUSION EQUATION WITH DRIFT

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0. Introduction

This paper is devoted to the growing up problem of a semi-linear diffusion equation

$$(1) \qquad u_t = u_{xx} + k(x)u_x + F(u) \qquad t>0,\ x>0$$

with the initial-boundary conditions

$$(2) \qquad \lim_{x \downarrow 0} e^{B(x)} u_x(t, x) = 0 \qquad t>0,$$

$$(3) \qquad \lim_{t \downarrow 0} u(t, x) = f(x)^{(1)} \qquad x>0$$

where $0 \leqq f(x) \leqq 1$ and we write

$$B(x) = \int_1^x k(y)dy \qquad x>0;$$

i.e. the problem of finding criteria of whether a solution u of (1)-(3) grows up or fades away. Here and henceforth a solution u of (1)-(3) is said to *grow up* if

$$(4) \qquad \lim_{t \to \infty} u(t, x) = 1 \qquad \text{locally uniformly in } x>0,$$

and to *fade away* if

$$(5) \qquad \lim_{t \to \infty} u(t, x) = 0 \qquad \text{uniformly in } x>0.$$

In treating of this problem, we are mainly interested in the case that the initial function f is zero out-side a finite interval. We will call such f which is not identical to zero a *finite initial function* (abbreviated to f.i.f.).

Throughout this paper it will be assumed that $F(u)$ and $k(x)$ are real valued, continuous and continuously differentiable functions on $0 \leqq u \leqq 1$ and on $x>0$, respectively, and that they satisfy the following conditions

(1) The convergence in (3) is taken in the locally L_1 sense.