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Osaka J. Math.
17 (1980), 177-186

REMARKS ON THE REGULARITY OF BOUNDARY POINTS IN A RESOLUTIVE COMPACTIFICATION

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(Received January 8, 1979)

Introduction. Let X be a strong harmonic space in the sense of Bauer [2] and suppose that constant functions are harmonic. In the previous paper [5], the author studied the regularity of boundary points in a resolutive compactification of X and discussed characterization of regularity, existence of regular points, strong regularity and pseudo-strong regularity, characterization of harmonic boundary and consideration in the case of open subsets. In this paper we shall use the same notations and definitions as in [5], and we shall give some supplementary remarks.

In §1, we recall the notations and terminologies used in [5]. We reform characterization of the regularity in Theorem 1 of §2. Theorem 2 in §3 is the extremal characterization of pseudo-strong regularity in the case where X is a Brelot space. The trace filters of neighborhoods of boundary points in the Wiener compactification X^W of X is of some interest. Using this filters we can construct in §4 a family of completely regular filters in a metrizable and resolutive compactification X^* of X. A regular boundary point x is said to have a local property if x is regular for every $\overline{U(x)\cap X}$, where $U(x)$ is a neighborhood of x. The main results of this paper are in §5. It is shown that a regular point x does not possess a local property in general and x has a local property if and only if x is pseudo-strongly regular. Further the related problems are investigated. In the final section, we consider a relatively compact open set G of a Brelot space and obtain the result, if G is minimally bounded, then the set of all regular points is dense in the boundary ∂G of G, which is a generalization of a result of Bauer [1].

1. Preliminaries

Let X be a *strong* harmonic space in the sense of Bauer [2] on which constant functions are harmonic, and X^* be a resolutive compactification of X. On the boundary $\Delta=X^*\backslash X$ we define the harmonic boundary $\Gamma=\{x\in\Delta;\ \underline{\lim}_{a\to x}p(a)=0$ for every strictly positive potential p on $X\}$. For $f\in C(\Delta)$, i.e., a continuous