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THE AUTOMORPHISM GROUP AND THE SCHUR MULTIPLER OF THE SIMPLE GROUP OF ORDER $2^{14}\cdot 3^6\cdot 5^6\cdot 7\cdot 11\cdot 19$

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As in [1], F denotes the simple group of order $2^{14}\cdot 3^6\cdot 5^6\cdot 7\cdot 11\cdot 19$. F is popularly called F_5 as it appears in the centralizer of an element of order 5 of the so called "Monster."

The simple group F has been constructed by S. Norton [2] and the automorphism group of it has also been determined by him. From his construction of F it can be seen that F has an outer automorphism of order 2.

In this note, we shall give an alternate proof of the fact that $|\mathrm{Aut}(F)\colon F|\leq 2$. We also show that the Schur multiplier of F is trivial.

Theorem A. $|\mathrm{Aut}(F)\colon F|=2$ and $H^2(F, C^*)=0$.

By [1, Proposition 2.13], F contains a subgroup F_0 isomorphic to the alternating group A_{12} of degree 12. It is easy to see the following:

Lemma 1. *F_0 is maximal in F. Every subgroup of F isomorphic to F_0 is conjugate in F to F_0.*

Proof of the first part of Theorem A. Suppose that $|\mathrm{Aut}(F)\colon F|>2$. Then there exists an element $\alpha\in\mathrm{Aut}(F)$ of order p, p a prime, such that $C_F(\alpha)\supseteq F_0$. Let x be an element of $F_0\cong A_{12}$ of type (12345). Then by [1, Lemma 2.17], $C_F(x)\cong Z_5\times U_3(5)$. Since no element of $\mathrm{Aut}(U_3(5))^\#$ centralizes a subgroup of $U_3(5)$ isomorphic to A_7, $\langle C_F(x), \alpha\rangle\cong\langle\alpha\rangle\times Z_5\times U_3(5)$. Hence by the maximality of F_0, $[F,\alpha]=1$. This contradiction shows that $|\mathrm{Aut}(F)\colon F|\leq 2$.

Proof of the second part of Theorem A. Let $m(F)$ be the order of the Schur multiplier of F. We denote by $m_p(F)$ the p-part of $m(F)$. $\tilde{F}$ will denote a central extension of F. For a subgroup A of F, $\tilde{A}$ will denote the inverse image of A in $\tilde{F}$.

Lemma 2. $m_2(F)=1$.

Proof. Let $\tilde{F}$ be a group such that $\tilde{F}/Z(\tilde{F})\cong F$ and $Z(\tilde{F})\cong Z_2$. F contains

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