Hiramine, Y.
Osaka J. Math.
15 (1978), 613-631

ON DOUBLY TRANSITIVE PERMUTATION GROUPS

YUTAKA HIRAMINE

(Received April 22, 1977)

1. Introduction

Let G be a doubly transitive permutation group on a finite set Ω and $\alpha\in\Omega$. Using the notation of [9], we denote a normal subgroup of G_α by N^α. Then, for $\beta\in\Omega$ other, we define N^β so that $g^{-1}N^\beta g=N^\gamma$ where $\gamma=\beta^g$.

In this paper we shall prove the following:

Theorem 1. *Let G be a doubly transitive permutation group on a finite set Ω. Suppose that α is an element of Ω. If G_α has a normal simple subgroup N^α which is isomorphic to $PSL(2, q)$, $Sz(q)$ or $PSU(3, q)$ with $q=2^n$, $n\geq 2$, then one of the following holds*:

(i) $|\Omega|=6$, $G\simeq A_6$ *or* S_6 *and* $N^\alpha\simeq PSL(2, 4)$.

(ii) $|\Omega|=11$, $G\simeq PSL(2, 11)$ *and* $N^\alpha\simeq PSL(2, 4)$.

(iii) *G has a regular normal subgroup.*

We introduce some notations: Let G be a permutation group on Ω. For $X\leq G$ and $\Delta\subseteq\Omega$, we define $F(X)=\{\alpha\in\Omega\,|\,\alpha^x=\alpha$ for all $x\in X\}$, $X(\Delta)=\{x\in X\,|\,\Delta^x=\Delta\}$, $X_\Delta=\{x\in X\,|\,\alpha^x=\alpha$ for all $\alpha\in\Delta\}$ and $X^\Delta=X(\Delta)/X_\Delta$, the restriction of X on Δ. If p is a prime, we denote by $O^p(X)$, the subgroup of X generated by all p'-elements in X. Other notations are standard ([6], [16]).

2. Preliminary results

Lemma 2.1. *Let G be a doubly transitive permutation group on Ω of even degree and N^α a nonabelian simple normal subgroup of G_α with $\alpha\in\Omega$. If $C_G(N^\alpha)\neq 1$, then $N^\alpha_\beta=N^\alpha\cap N^\beta$ for $\alpha\neq\beta\in\Omega$ and $C_G(N^\alpha)$ is semi-regular on $\Omega-\{\alpha\}$.*

Proof. Set $C^\alpha=C_G(N^\alpha)$. By Corollary B3 and Lemma 2.8 of [17], C^α is semi-regular on $\Omega-\{\alpha\}$ or N^α is a T.I. set in G. Since $|\Omega|$ is even and N^α is $\frac{1}{2}$-transitive on $\Omega-\{\alpha\}$, $|N^\alpha: N^\alpha_\beta|$ is odd for $\alpha\neq\beta\in\Omega$. Hence N^α is not semiregular on $\Omega-\{\alpha\}$. By Theorem A of [9], N^α is not a T.I. set in G. Hence C^α is semi-regular on $\Omega-\{\alpha\}$.

Set $\Delta=F(N^\alpha_\beta)$. Since $C^\alpha\leq G(\Delta)$, $[C^\alpha, G_\Delta]\leq C^\alpha\cap G_\Delta=1$. By Corollary