Iwata, K.
Osaka J. Math.
15 (1978), 475–508

CLASSIFICATION OF COMPACT TRANSFORMATION GROUPS ON COHOMOLOGY QUATERNION PROJECTIVE SPACES WITH CODIMENSION ONE ORBITS

KOICHI IWATA

(Received August 8, 1977)
(Revised February 16, 1978)

0. Introduction

Let M be an orientable closed $4n$-dimensional smooth manifold, whose rational cohomology algebra is isomorphic to that of a quaternion projective n-space $P_n(\boldsymbol{H})$. We call such a manifold M a rational cohomology quaternion projective n-space.

Let (G, M) be a pair of a compact connected Lie group G and a simply connected rational cohomology quaternion projective n-space M, on which G acts smoothly with a codimension one orbit G/K. We say that (G, M) is isomorphic to (G', M'), if there exist a Lie group isomorphism $h: G \to G'$ and a diffeomorphism $f: M \to M'$ satisfying

$$f(gx) = h(g)f(x),$$

for every $g \in G$ and for every $x \in M$.

When G acts on M, $H = \bigcap_{x \in M} G_x$ (the intersection of all isotropy groups) is a closed normal subgroup of G. Since H acts on M trivially, the G-action on M induces an effective G/H-action on M. We say that (G, M) is essentially isomorphic to (G', M'), if there exists an isomorphism between the pairs with effective actions $(G/H, M)$ and $(G'/H', M')$.

The purpose of the present paper is to give a complete classification of such pairs (G, M) up to essential isomorphism. We shall show

Main Theorem. *Such a pair (G, M) is essentially isomorphic to one of the pairs listed in the next table.*