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ON THE SCHUR INDICES OF THE FINITE UNITARY GROUPS

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Introduction

Let $\boldsymbol{F}_q$ denote the finite group of characteristic p with q elements. We consider the finite unitary group $U(n, q^2)$ of rank n relative to the quadratic extension $\boldsymbol{F}_{q^2}/\boldsymbol{F}_q$. For a complex irreducible character χ of a finite group, the Schur index of χ with respect to the field $\boldsymbol{Q}$ of rational numbers is defined to be the minimal degree among all the extensions $K/\boldsymbol{Q}(\chi)$ such that χ is realizable in K. Here $\boldsymbol{Q}(\chi)$ is the extension of $\boldsymbol{Q}$ generated by the values of χ. We denote this index by $m_{\boldsymbol{Q}}(\chi)$. In this paper, we shall determine the Schur indices of all the complex irreducible characters of $U(n, q^2)$ for sufficiently large p and q. Our main result is the following theorem.

Main Theorem. *Assume that p and q are sufficiently large. Then the Schur index of any complex irreducible character of $U(n, q^2)$ with respect to the field of rational numbers is one.*

REMARK. If $n \leq 5$, it is enough only to assume $p \neq 2$ (see §2).

The theorem follows from

Theorem A (R. Gow [3], p 112). *For any complex irreducible character χ of $U(n, q^2)$, $m_{\boldsymbol{Q}}(\chi)$ divides 2.*

Theorem B. *The values of any complex irreducible character of $U(n, q^2)$ on unipotent elements are rational integers and its Schur index divides these values.*

This will be proved in Section 4.

Theorem C. *Assume that p and q are sufficiently large (if $n \leq 5$, this assumption can be dropped out). Then for any complex irreducible character χ of $U(n, q^2)$, there is a unipotent element u of $U(n, q^2)$ such that $\chi(u)$ is equal to the p-part of the degree of χ up to sign.*

This will be proved in Section 2.