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Osaka J. Math.
15 (1978), 151–159

ON THE ALEXANDER POLYNOMIALS OF COBORDANT LINKS

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(Received November 26, 1976)
(Revised September 1, 1977)

R.H. Fox and J.W. Milnor in [4] showed that the Alexander polynomial of a slice knot is of the form $f(t)f(t^{-1})$ for an integral polynomial $f(t)$ with $|f(1)|=1$. This clearly implies that the Alexander polynomials of cobordant knots are identical up to the polynomials of the form $f(t)f(t^{-1})$. The purpose of this paper is to generalize this property to that of arbitrary cobordant links. On the basis of the work done by K. Reidemeister, H.G. Shumann and W. Burau, R.H. Fox defined the μ-variable Alexander polynomial $A^0(t_1, \cdots, t_\mu)$ of a link L^μ with μ components. (cf. R.H. Fox [3], G. Torres [9].) One difficulty in our study is that using this definition the polynomial $A^0(t_1, \cdots, t_\mu)$ vanishes for many links. For example, any decomposable link (that is, a link separated into two sublinks by a 2-sphere within a 3-sphere) has $A^0(t_1, \cdots, t_\mu)=0$. To avoid this difficulty we shall re-define the Alexander polynomial $A(t_1, \cdots, t_\mu)$ so that it is *always a non-zero polynomial*. To measure the difference between $A_0(t_1, \cdots, t_\mu)$ and $A(t_1, \cdots, t_\mu)$, we will also introduce a numerical invariant $\beta(L^\mu)$ with $0\leq\beta(L^\mu)\leq\mu-1$ such that

$$A^0(t_1, \cdots, t_\mu) = \begin{cases} A(t_1, \cdots, t_\mu) & \text{if } \beta(L_\mu)=0 \\ 0 & \text{if } \beta(L^\mu)\neq 0\,. \end{cases}$$

A link is the disjoint union of piecewise-linearly embedded, oriented 1-spheres in the oriented 3-sphere S^3. Two links L_0 and L_1 with μ components are *PL cobordant,* if there exist mutally disjoint, piecewise-linearly embedded proper annuli $F_1, \cdots, F_\mu$ in $S^3\times[0, 1]$ spanning $S^3\times 0$ and $S^3\times 1$ such that $(F_1\cup\cdots\cup F_\mu)\cap S^3\times 0=L_0\times 0$ and $(F_1\cup\cdots\cup F_\mu)\cap S^3\times 1=(-L_1)\times 1$, where $-L_1$ is L_1 with orientation reversed. If the annuli $F_1, \cdots, F_\mu$ are locally flat, then the links L_0 and L_1 are simply said to be *cobordant.* A link that is cobordant to the trivial link is called *a slice link in the strong sense.* (cf. R.H. Fox [3].) For (PL) cobordant links L_i, $i=0, 1$ with μ components the Alexander polynomials $A_i(t_1, \cdots, t_\mu)$ of L_i should be chosen to be the Alexander polynomials associated with meridian bases of $H_1(S^3-L_i; Z)$ consistent through the cobordism annuli $F_1, \cdots, F_\mu$.