

ADMISSIBLE REPRESENTATIONS FOR SPLIT REDUCTIVE GROUPS DEFINED OVER A FUNCTION FIELD*

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1. Introduction

In this paper we shall study the admissibility of representations for a reductive group defined over a global field k of characteristic $p \neq 0$. For the concept of admissible representations in the sense used here and their connection with the Siegel Formula for number fields see [7]. and [11].

We obtain a necessary and sufficient condition for the convergence of the integral $I_\rho(\Phi)$ associated with the representation ρ . This criterion of convergence involves the weights and their multiplicities in ρ and is analogous to Weil's result for number fields [9 p. 20]. We see that although representations need no longer be completely reducible, the admissibility of the triple (G, X, ρ) depends only on the composition factors of ρ .

As a corollary, for G connected and reductive over k , we see that G_A/G_k has finite volume if and only if the centre of G has no k -split torus. In particular this implies that for G to have any non-trivial absolutely admissible representation over k , G is necessarily semi-simple. Further, we see that for given G only finitely many different composition series can occur as the composition series of admissible representations.

For G a simply connected, simple and k -split group we obtain a list of composition series that can occur in admissible representations. The list includes all the representations that occur for number fields, as well as many new ones, depending on the characteristic of k . For $p = \text{characteristic of } k$ sufficiently large we show the list to be complete. However, for p small we cannot conclude that the list is exhaustive. The difficulty is that for small p it is an unsolved problem as to what the weights and multiplicities of a given irreducible representation ρ_λ are when the highest weight λ is given.

We further show how to obtain the admissible representations when G is a simple group.

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