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AN APPLICATION OF THE THEORY OF DESCENT TO THE $S\otimes_R S$-MODULE STRUCTURE OF S/R-AZUMAYA ALGEBRAS

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Introduction. Let R be a commutative ring and S a commutative R-algebra which is a finitely generated faithful projective R-module. An R-Azumaya algebra A is called an S/R-Azumaya algebra if A contains S as a maximal commutative subalgebra and is left S-projective. S-S-bimodule structure (for which we shall call $S\otimes_R S$-module structure) of S/R-Azumaya algebras is determined in [5] when S/R is a separable Galois extension and in [8] when S/R is a Hopf Galois extension, both are connected with one which is so called seven terms exact sequence due to Chase, Harrison and Rosenberg [3].

In this paper we shall investigate the $S\otimes_R S$-module structure of S/R-Azumaya algebras assuming only that S is a finitely generated faithful projective R-module. So S/R-Azumaya algebras are not necessarily $S\otimes_R S$-projective (c.f. [8] Th. 2.1). But in §1 we shall show for any S/R-Azumaya algebra A, there exists a unique finitely generated projective $S\otimes_R S$-module P of rank one with certain cohomological properties such that A is $S\otimes_R S$-isomorphic to $P\otimes_{S\otimes_R S}\mathrm{End}_R(S)$. In §2, we shall investigate S/R-Azumaya algebras resulting from Amitsur's 2-cocycles. Finally we shall deal with the seven terms exact sequence in §3.

Throughout R will be a fixed commutative ring with unit, a commutative R-algebra S is a finitely generated faithful projective as R-module, each $\otimes$, End, etc. is taken over R unless otherwise stated. Repeated tensor products of S are denoted by exponents, $S^q=S\otimes\cdots\otimes S$ with q-factors. We shall consider S^q as an S-algebra on first term. To indicate module structure, we write if necessary, $S_1\otimes S_2$ instead of $S^2=S\otimes S$, ${}_{S_1}M_{S_2}$ instead of $S^2=S_1\otimes S_2$-module M etc.. $H^q(S/R, U)$ and $H^q(S/R, Pic)$ denote the q-th Amitsur's cohomology groups of the extension S/R with respect to the unit functor U and Picard group functor Pic respectively.

1. S/R-Azumaya algebras and $H^1(S/R, Pic)$

First we prove the following, which clarify the S^2-module structure of