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DEGENERATE ELLIPTIC SYSTEMS OF PSEUDO-DIFFERENTIAL EQUATIONS AND NON-COERCIVE BOUNDARY VALUE PROBLEMS

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0. Introduction

In the present paper we shall study a class of degenerate elliptic systems of pseudo-differential equations, and apply the results obtained there to non-coercive boundary value problems of fourth order.

One of typical examples of non-coercive problems is the oblique derivative problem: Let Ω be a bounded open set in $\boldsymbol{R}^n$ with a smooth boundary Γ and consider the problem

$$(0.1)\qquad \begin{cases} A(x, D_x)u = f & \text{in } \Omega\,, \\ \dfrac{\partial u}{\partial \nu} = g & \text{on } \Gamma\,, \end{cases}$$

where $A(x, D_x)$ is an elliptic differential operator of second order on $\bar{\Omega}$ and ν is a non-vanishing real vector field tangent to Γ on its submanifold Γ_0. The behavior of ν near Γ_0 has a crucial effect on this problem (for details, see [5], [14], etc.). We shall consider in §4 a similar problem for an elliptic operator $L(x, D_x)$ of fourth order on $\bar{\Omega}$:

$$(0.2)\qquad \begin{cases} L(x, D_x)u = f & \text{in } \Omega\,, \\ \dfrac{\partial^2 u}{\partial \nu_1 \partial n} = g_1 & \text{on } \Gamma\,, \\ \dfrac{\partial u}{\partial \nu_2} = g_2 & \text{on } \Gamma\,, \end{cases}$$

where ν_1, ν_2 are vector fields of the same type as in (0.1). We study this problem by a usual method. Namely, let $\mathscr{P}$ be the Poisson operator of the Dirichlet problem

$$\begin{cases} L(x, D_x)u = f & \text{in } \Omega\,, \\ D_n u = h_1 & \text{on } \Gamma\,, \\ u = h_2 & \text{on } \Gamma\,, \end{cases}$$