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PROJECTIVE DIMENSION OF BROWN-PETERSON HOMOLOGY WITH MODULO $(p, v_1, \cdots, v_{n-1})$ COEFFICIENTS

Dedicated to the memory of Professor Taira Honda

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Let BP denote the Brown-Peterson spectrum for a fixed prime p. This spectrum gives us a multiplicative homology theory $BP_*(\ )$ defined on the category of CW-spectra, with coefficient ring $BP_*=Z_{(p)}[v_1, \cdots, v_n, \cdots]$. Recall that the only invariant prime ideals of BP_* are those of the form $I_n=(v_0, v_1, \cdots, v_{n-1})$, $0\leqq n\leqq\infty$, where we put $v_0=p$. Using Baas-Sullivan technique we can kill the generators $v_0, v_1, \cdots, v_{n-1}, v_{m+1}, v_{m+2}, \cdots$ of BP_* to construct a homology theory $BP[n, m+1)_*(\ )$ represented by a certain BP-module spectrum $BP[n, m+1)$, where $0\leqq n\leqq m+1\leqq\infty$. In particular $BP[n, \infty)$ is the Brown-Peterson spectrum with modulo I_n coefficients, denoted by $P(n)$ [4]. According to Morava's geometric computation, $P(n)_*(X)$ becomes a module over the coherent ring $P(n)_*=BP_*/I_n$. Morava and also Johnson-Wilson[4] have obtained rich results concerned with the BP-module spectrum $P(n)$ and its operations.

Define hom $\dim_{P(n)_*}M$ to be the projective dimension of M as $P(n)_*$-module. Johnson-Wilson[3] gave completely satisfactory conditions under which hom $\dim_{BP_*}BP_*(\mathrm{X})\leqq m$, where they assumed that X is a based finite CW-complex. The proof was based on Wilson's splitting theorem. Later Landweber[7] introduced the category $\mathcal{BP}_0$ of coherent comodules over $BP_*(BP)$ which has $BP_*(X)$ as object for every based finite CW-complex X. He then proved algebraically the result of Johnson-Wilson, applying two powerful tools called Filtration theorem and Exact functor theorem. The purpose of this note is to give the $P(n)_*(\ )$-version of the result of Johnson-Wilson without the finiteness assumption on X, *i.e.*, the characterization of numerical invariant hom $\dim_{P(n)_*}P(n)_*(X)$ for a connective CW-spectrum X (Theorem 4.8). In the proof we will use Landweber's methods.

Let X be a connective CW-spectrum. Using structure theorem of $P(n)^*(P(n))$ we can see easily that $P(n)_*$-modules $P(n)_*(X)$ become comodules over $BP_*(BP)$ when $n<2(p-1)$ (see [4, Remark 2.14]). But the author