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K-GROUPS OF SYMMETRIC SPACES II

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1. Introduction

Let $M=G/K$ be a symmetric homogeneous space such that G is a simply connected compact Lie group. In [I] the author showed that the unitary K-group of M is isomorphic to the tensor product of $R(K)\underset{R(G)}{\otimes}Z$ and an exterior algebra E over Z, where $R(G)$ and $R(K)$ are the complex representation rigns of G and K respectively, and in particular described the generators of E as an exterior algebra explicitly.

The purpose of this paper is to present a structure of $R(K)\underset{R(G)}{\otimes}Z$ as a group in the following nine cases:

$$\text{Type of } M = AIII,\ BDI(a)(Spin(2p+2q+2)/Spin(2p+1)\cdot Spin(2q+1)),$$
$$BDII(b)(Spin(2n+1)/Spin(2n)),\ DIII,\ CII,\ EI,\ FI,\ FII \text{ or } G.$$

Now let us denote by $n(L)$ the order of the Weyl group of a compact connected Lie group L. We know that if U is a closed connected subgroup of G of maximal rank then $R(U)\underset{R(G)}{\otimes}Z$ is a free module of rank $n(G)/n(U)$ and is isomorphic to $K^*(G/U)$ [12]. Throughout this paper we shall identify $R(U)\underset{R(G)}{\otimes}Z$ with the K-group of G/U in the above situation and denote by the same letter ρ the element of $K^*(G/U)$ defined by an element ρ of $R(U)$ in the natural way. Furthermore we shall denote by $Z(g)$ the free module generated by an element g.

2. Representation rings

In this section we recall the structure of the complex representation rings of classical groups.

Write ρ_n for the canonical representations $SU(n)\to U(n)$, $U(n)\to U(n)$, $Sp(n)\to U(2n)$ and $Spin(n)\to SO(n)\to U(n)$ for each n, and write $\lambda^i\rho_n$ for the i-th exterior product of ρ_n. According to [10] we have