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# SOME NILPOTENT H-SPACES

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## 0. Introduction

In this note we give two generalisations, (Proposition 1.2 & Theorem 1.3), of Stasheff's criterion for homotopy commutativity of $H$-spaces, [11, Theorem 1.9], and apply them to produce examples of nilpotent $H$-spaces and to demonstrate the vanishing of certain Samelson-Whitehead products.

In §1.2 we give a necessary and sufficient condition for the vanishing of the Samelson-Whitehead product of $f:SA\to Y$ and $g:SB\to Y$. In Theorem 1.3 a criterion for the vanishing of the $j$-th iterated commutator map in an $H$-space, $X$, is given in terms of a space, $X(j)$. As a corollary it is shown that if the projective plane of $X$, (resp. the space $X$), has a finite Postnikov system then $X$, (resp. $\Omega X$), is nilpotent. In §2 the nilpotency of loop spaces of spheres and projective spaces is discussed. Many of the results of §2 are known to other authors and I am grateful to G.J. Porter for drawing my attention to the results of T. Ganea, [3]. However, for completeness, the results of [3] have been included here, as corollaries of Proposition 1.2. The nilpotency of $\Omega S^{2n}$ and $\Omega CP^{2n}$ do not appear in [3] although the former was previously known to M.G. Barratt, I. Berstein and T. Ganea. Since our estimate of the nilpotency of $\Omega CP^{2n}$ is large we include a corollary of Theorem 1.3 on the vanishing of a family of triple Samelson-Whitehead products on $CP^{2n}$.

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In this paper we work in the category of based, countable $CW$ complexes. A connected complex in this category is called special. The following notation is used:—

$X\wedge Y$=smash product of $X$ and $Y$.,

$\overset{j}{\vee}X$, $\overset{j}{\wedge}X$ and $X^j$ are respectively the $j$-fold wedge, smash and product of $X$,

$I=[0, 1]$ with basepoint, $*=0$,

$SX=S^1\wedge X$, $\Omega X$=the space of loops on $X$,

and (eval: $S\Omega X\to X$)=the evaluation map.