Bannai, E.
Osaka J. Math.
13 (1976), 123–129

ON MULTIPLY TRANSITIVE PERMUTATION GROUPS IV

EIICHI BANNAI*)

(Received February 12, 1975)

Introduction

By combining the results of Miyamoto [5] and Bannai [1, 2], we have obtained the following theorem ([2, Main Theorem]) which is an odd prime version of a theorem of M. Hall [3].

Theorem. Let p be an odd prime. Let G be a $2p$-*ply* transitive permutation group such that $G_{1,2,\cdots,2p}$ (=the pointwise stabilizer of $2p$ points) is of order prime to p. Then G is one of $S_n(2p\leq n\leq 3p-1)$ and $A_n(2p+2\leq n\leq 3p-1)$, where S_n and A_n denote the symmetric and alternating groups of degree n.

The purpose of this paper is to generalize the above theorem. Namely, we will prove the following theorem.

Theorem 1. *Let p be an odd prime. Let G be a $2p$-ply transitive permutation group such that either*
(i) *each element in G of order p fixes at most $2p+(p-1)$ points, or*
(ii) *a Sylow p subgroup of $G_{1,2,\cdots,2p}$ is cyclic.*
Then G is one of $S_n(2p\leq n\leq 4p-1)$ and $A_n(2p+2\leqq n\leq 4p-1)$.

Note that Theorem 1 (i) and Theorem 1 (ii) are some odd prime versions of a theorem of Nagao [6] and a theorem of Noda and Oyama [7] respectively.

The essential part of the proof of Theorem 1 (i) is picked up as follows:

Theorem A. *Let p be an odd prime. Then there exists no $(p+3)$-ply transitive permutation group G on a set $\Omega=\{1,2,\cdots,n\}$ which satisfies the following two conditions:*
(1) *a Sylow p subgroup $P(\neq 1)$ of $G_{1,2,\cdots,p+3}$ fixes at most $p-1$ points in $\Omega-\{1,2,\cdots,p+3\}$, and P is semiregular on $\Omega-I(P)$, where $I(P)$ denotes the set of the points which are fixed by any element of P.*
(2) $|\Omega-I(P)|\not\equiv p \pmod{p^2}$.

Note that Theorem A generalizes Lemma 1.5 in Miyamoto [5] to some

*) Supported in part by the Sakkokai Foundation.