Ogura, Y. and Shiotani, K.
Osaka J. Math.
13 (1976), 83–98

ON INVARIANT MEASURES OF CRITICAL MULTITYPE GALTON-WATSON PROCESSES

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(Received September 28, 1974)

1. Introduction

A multitype Galton-Watson process with discrete time (abbreviated to MGWP) is a mathematical description of a population growth involving several types of individuals, where each individual produces offspring by a certain stochastic law independently of others. Suppose that one i-type individual produces x^1 individuals of 1-type, x^2 ones of 2-type, $\cdots$, x^N ones of N-type during a unit time with probability $P^i(x)$ ($x=(x^1, \cdots, x^N)$). Then an MGWP is defined as a Markov chain on the space S of all N-tuples of nonnegative integers, with the one step transition probability

$$(1.1) \qquad P(x, y) = \begin{cases} \underbrace{P^1 * \cdots * P^1}_{x^1} * \cdots * \underbrace{P^N * \cdots * P^N}_{x^N}(y)\,, & \\ \qquad\qquad \text{if } 0 \neq x = (x^1, \cdots, x^N) \in S,\, y \in S\,, & \\ \delta_0(y), \qquad \text{if } x = 0 \in S,\, y \in S\,, & \end{cases}$$

where $*$ means the convolution of distributions. Since the state $0 \in S$ is a trap for our process, invariant measures on the whole state space S are trivial in most cases. But invariant measures of the MGWP restricted onto $S-\{0\}$ are not trivial in general, and it is important to study them. For the case of $N=1$, many authors have investigated this subject (cf. Harris [4] pp. 22–31, Athreya and Ney [2] pp. 67–73, 87–93]. Especially, Kesten, Ney and Spitzer [7] gave the definitive results on the existence and uniqueness of invariant measures of critical simple GW processes.

In this paper, we shall prove the existence and uniqueness of invariant measures on $S-\{0\}$ of a critical, positively regular and nonsingular MGWP, under the hypothesis of finite $x^2\log x$-moments (cf. (H.1)$\sim$(H.4) in §2). The statement of the theorem is given in section 2. In section 3 we shall prove a basic lemma which was proved in [7] ((2.16), p. 517) in the case of critical simple GW processes. It will be proved by elaborating those results in [5] since the proof in [7] does not seem applicable to the case of multitype GW processes. Finally, in section 4, we shall prove the theorem with the aid of the basic lemma.