

ON THE NONLINEAR DIFFUSION EQUATION OF KOLMOGOROV-PETROVSKII-PISKUNOV TYPE

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Introduction

Kolmogorov, Petrovskii and Piskunov [1], whom we shall refer to as KPP, studied the initial value problem for a semilinear diffusion equation

$$(1) \quad \begin{cases} Lu = f(u), & 0 \leq u \leq 1 & (x, t) \in R^1 \times (0, \infty), \\ u(x, 0) = u_0(x) & x \in R^1, \\ \left(\text{where } L = \frac{\partial}{\partial t} - \left(\frac{\partial}{\partial x} \right)^2, \quad u = u(x, t), \right) \end{cases}$$

under the conditions:

- (i) $f(\xi) \in C^\infty[0, 1]$,
- (ii) $f(0) = f(1) = 0, \quad f'(0) > 0 > f'(1)$,
- (iii) $f(\xi) > 0, \quad f_0(\xi) \equiv f'(0)\xi - f(\xi) \geq 0 \quad \xi \in (0, 1).$