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ON STABLE JAMES NUMBERS OF QUATERNIONIC PROJECTIVE SPACES

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In [4], we have defined the stable James numbers $k_s(X, A)$ for some finite CW-pairs (X, A) and computed $d_C(n)=k_s(CP^n, CP^1)$. In this note we estimate $d_H(n)=k_s(HP^n, HP^1)$, where HP^n denotes the quaternionic projective space of topological dimension $4n$. We obtain

Theorem. *For $n\geqq 2$*

(0) *$d_H(n)$ is a factor of $(2n)!\,(2n-2)!\cdots 4!$, in particuler none of the prime factors of $d_H(n)$ is greater than $2n$,*

$$\text{(i)}\quad 2j+1\leqq\begin{cases} \nu_2(d_H(n))\leqq 3j+3 & \textit{for}\quad n=2^j, \\ \nu_2(d_H(n))\leqq 3j+6 & \textit{for}\quad 2^j+1\leqq n<2^{j+1}, \end{cases}$$

$$\text{(ii)}\quad 2j\leqq\nu_3(d_H(n))\leqq 2j+2 \qquad \textit{for}\quad 3^j\leqq n<2\cdot 3^j,$$

$$2j+1\leqq\begin{cases} \nu_3(d_H(n))\leqq 2j+2 & \textit{for}\quad 2\cdot 3^j\leqq n<\dfrac{3^{j+2}+1}{4}, \\ \nu_3(d_H(n))\leqq 2j+4 & \textit{for}\quad \dfrac{3^{j+2}+1}{4}\leqq n<3^{j+1}, \end{cases}$$

(iii) *for a prime $p\geqq 5$*

$$\nu_p(d_H(n))=2j \qquad \textit{for}\quad p^j\leqq n<\frac{p^{j+1}+1}{4},$$

$$2j\leqq\nu_p(d_H(n))\leqq 2j+2 \quad \textit{for}\quad \frac{p^{j+1}+1}{4}\leqq n<\frac{p+1}{2}p^j,$$

$$2j+1\leqq\nu_p(d_H(n))\leqq 2j+2 \quad \textit{for}\quad \frac{p+1}{2}p^j\leqq n<p^{j+1},$$

where $\nu_p(m)$ denotes the exponent of p in the prime factorization of m.

Recall that $d_H(n)$=the index of the image of i^*: $\{HP^n, S^4\}\to\{S^4, S^4\}$, where $\{X, Y\}$ denotes the set of stable homotopy classes of stable maps $X\to Y$ and i: $S^4=HP^1\to HP^n$ the natural inclusion. Then obviously $d_H(1)=1$.