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SOME RELATIONS AMONG VARIOUS NUMERICAL INVARIANTS FOR LINKS

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Introduction. Throughout this paper, "*a link l of $\mu(l)$ components*" means disjoint union of $\mu(l)$ oriented 1-spheres in R^3.

In §1, we study some 3-dimensional numerical invariants of links, that is, $g(l)$ (genus of l), $u(l)$ (see Definition 1) and $c(l)$ (see Definition 3) will be defined and we will have some relations among them as follows.

Theorem 1. *For any link l, $g(l)\leqq c(l)$ and $u(l)\leqq c(l)$.*

In §2, the 4-dimensional numerical invariants $g^*(l)$, $g_r^*(l)$ (see Definition 4), $u^*(l)$, $u_r^*(l)$ (see Definition 5), $c^*(l)$ and $c_r^*(l)$ (see Definition 6) will be defined and the main theorem will be proved.

Theorem 2. *For any link l, we obtain*

$$\begin{array}{ccccc} g^*(l) & \leqq & g_r^*(l) & \leqq & g(l) \\ \wedge\!\!\parallel & & \wedge\!\!\parallel & & \wedge\!\!\parallel \\ c^*(l) & \leqq & c_r^*(l) & \leqq & c(l) \\ \wedge\!\!\parallel & & \parallel & & \vee\!\!\parallel \\ u^*(l) & \leqq & u_r^*(l) & \leqq & u(l) \end{array}$$

As is usual, two links l and l' are said to be of the *same type* or *isotopic*, denoted by $l\approx l'$, if there exists an orientation preserving homeomorphism f of R^3 onto itself such that $f(l)=l'$.

∂X, *Int* X and *cl* X represents the *boundary*, the *interior* and the *closure* of X respectively.

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1. 3-dimensional numerical invariants

Let l be a link of $\mu(l)$ components in R^3. It is known in [9], [11] that l always bounds an orientable connected surface F in R. The minimum genus of these surfaces is called the *genus* of the link l and is denoted by $g(l)$. Note that $g(F)$ denotes the usual genus of a surface F.