Marubayashi, H.
Osaka J. Math.
9 (1972), 231-259

ON POTENT RINGS AND MODULES

Hidetoshi MARUBAYASHI

(Received April 19, 1971)

There are two kinds of "quotient ring". One is called a classical quotient ring, that is, an extension ring $Q(R)$ of a ring R is called an classical right quotient ring of R if

(i) $Q(R)\ni 1$,

(ii) every element of $Q(R)$ has the form ac^{-1}, where $a, c\in R$ and c is a regular element of R,

(iii) every regular element of R has an inverse in Q.

In [6], [7], [19], [20] and [21] etc., many authors studied the structure of those rings which have an artinian classical right quotient ring. Such rings have finite dimensions in the sense of Goldie. It seems to the author that there does not exist too many rings with infinite dimensions which have the classical right quotient ring (even when the right singular ideal of such rings vanishes).

The other quotient ring is called a (homological) quotient ring and was defined by R. E. Johnson [10], Y. Utumi [22], G. D. Findlay and J. Lambek [5]. An extension ring S of a ring R is a right quotient ring of R if for each a, $0\neq b\in S$, there exist $r\in R$ and $n\in Z$ such that $ar+na\in R$ and $br+nb\neq 0$, where Z is the ring of integers. If R is a left faithful ring, then R has a unique maximal right quotient ring $\hat{R}$. In particular, if R has zero right singular ideal, then $\hat{R}$ is a right self-injective von Neumann regular ring. So when we investigate rings with zero right singular ideal, it is useful to consider the (homological) maximal right quotient rings of such rings. But a ring R need not be semi-prime even in the case where $\hat{R}$ is simple and artinian, as the following example shows. Let D be a right Ore domain and let F be the right quotient division ring of D. We put

$$R=\left\{\left.\begin{bmatrix} a_{11} & 0\cdots 0 \\ a_{21} & 0\cdots 0 \\ \vdots & \vdots \quad \vdots \\ a_{n1} & 0\cdots 0\end{bmatrix}\right| a_{i1}\in D\right\} \text{ and } \hat{R}=(F)_n\,.$$

Then $\hat{R}$ is the maximal right quotient ring of R. The above example suggests that there are even various those rings which have the simple artinian maximal