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ON REALIZATION OF KIRBY-SIEBENMANN'S OBSTRUCTIONS BY 6-MANIFOLDS

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1. Introduction

Let M^n be a closed topological manifold. By Kirby-Siebenmann ([5], [6]), an obstruction to triangulate M^n is defined as an element of $H^4(M^n : Z_2)$, provided $n \geqslant 5$. We will denote this obstruction by $k(M)$. In this paper, we will consider the following problem.

Problem. Let M_0^n be a closed PL manifold. For a given non-zero element $\eta \in H^4(M_0^n : Z_2)$, do there exist a nontriangulable manifold M^n and a homotopy equivalence $f : M_0^n \rightarrow M^n$ such that $f^*k(M^n)=\eta$? Here, $f^* : H^4(M^n : Z_2) \rightarrow H^4(M_0^n : Z_2)$ is the isomorphism induced by f.

Since there exists a non-triangulable manifold M^6 which is homotopy equivalent to $S^4 \times S^2$ ([5], Introduction p.v), this problem for $M_0^n = S^4 \times S^2$ has an affirmative answer. In some cases, however, the problem has a negative answer. For example, Dr. S. Fukuhara has proved the following ([3]); let M^5 be a closed (possibly non-triangulable) topological manifold which is homotopy equivalent to $S^4 \times S^1$, then M^5 is really homeomorphic to $S^4 \times S^1$.

When M_0^6 is a closed manifold with $\pi_1(M_0^6)$ is free and $H^3(M_0^6 : Z_2)=0$, the problem will be answered affirmatively. And the problem for $M_0^n = S^4 \times S^{n-4}$ will be solved, provided $n \geqslant 9$. (See Corollary 2.)

The method of this paper can be found in [5] and [9]. The author wishes to express his hearty thanks to Professor K. Kawakubo who showed him a construction of non-triangulable manifold having the homotopy type of CP^3.

2. Six-dimensional case

In dimension six, our results are as follow.

Theorem 1. *Let M_0^6 be a closed PL 6-manifold with $H^3(M_0^6 : Z_2)=0$ and η a non-zero element of $H^4(M_0^6 : Z_2)$ whose Poincaré dual $\bar{\eta}$ is spherical. Then there exist a non-triangulable manifold M^6 and a homotopy equivalence $f : M_0^6 \rightarrow M^6$ such that $f^*k(M)=\eta$, where $f^* : H^4(M^6 : Z_2) \rightarrow H^4(M_0^6 : Z_2)$ is the isomorphism*