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THE CHARACTERS OF THE FINITE SYMPLECTIC GROUP Sp(4,q), q=2^f

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The purpose of this note is to calculate all the (complex) irreducible characters of the finite symplectic group $Sp(4, q)$ where $q=2^f$, while B. Srinivasan [3] determined the character table of $Sp(4, q)$ for odd q. All the irreducible characters of $Sp(4, 2^f)$ are expressed as linear combinations of induced characters with integral coefficients. The conjugacy classes of various subgroups are easily determined by the same method described in [1], and only the results are given (cf. [4]). For purposes of convenience the character tables of various subgroups (and of $Sp(4, 2^f)$ itself) are given in the Appendix.

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By similar but a little more complicated calculations, the author has obtained the character table of the finite Chevalley group $G_2(2^f)$, which will appear elsewhere.

1. Notation and preliminary results

Let K be the finite field with q elements, where $q=p^f$ and p is a prime number. Let $\bar{K}$ be the algebraic closure of K, and put

$$K_i = \{x\in\bar{K} \mid x^{q^i} = x\} .$$

Then K_i is the subfield of $\bar{K}$ with q^i elements, and $K_1=K$. Let κ be a fixed generator of the multiplicative group K_4^*, and put $\tau=\kappa^{q^2-1}$, $\theta=\kappa^{q^2+1}$, $\eta=\theta^{q-1}$ and $\gamma=\theta^{q+1}$. Then we have $\langle\theta\rangle=K_2^*$ and $\langle\gamma\rangle=K^*$. Choose a fixed isomorphism from the multiplicative group K_4^* into the multiplicative group of complex numbers, and let $\tilde{\tau}$, $\tilde{\theta}$, $\tilde{\eta}$ and $\tilde{\gamma}$ be the images of τ, θ, η and γ respectively under this isomorphism.

Let G be the 4-dimensional symplectic group over K, that is,

$$G = \{A\in GL(4, K) \mid {}^tAJA = J\} ,$$