

ON PRIME IDEALS AND PRIMARY DECOMPOSITIONS IN A NONASSOCIATIVE RING

HYO CHUL MYUNG

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1. Introduction

Prime ideals in nonassociative rings have been defined by a number of authors in different terms (for example, [1], [4], and [5]). In [5], the Brown-McCoy type prime ideals and radical have been defined for Jordan rings by using the quadratic operation. In [4], using $*$ -operation defined on the family of ideals and a function defined on the ring, the results in [5] have been extended to weakly W -admissible rings which generalize many of the well known nonassociative rings, in particular, alternative and Jordan rings.

For the associative case, the concept of prime ideals in the sense of McCoy [2] was generalized in [3] by defining f -systems which generalize the m -systems of McCoy. Also, f -primary ideals are defined in [3] and an analogue of the uniqueness theorem of the Lasker-Noether decomposition in the commutative case is proved for arbitrary associative rings in terms of f -primary ideals.

The essential purpose of this paper is to extend some of the results in [3] for associative rings to arbitrary nonassociative rings. Using the same function f as in [3] and the $*$ -operation, we give a definition of f -prime ideals for arbitrary nonassociative rings. Under certain choices of the $*$ -operation and the function f , our present f -prime ideals coincide with the prime ideals in [5] for Jordan rings and those in [3] for associative rings. We also obtain analogous results of f -primary decomposition in [3] for nonassociative rings.

2. f -prime ideals

Let R be an arbitrary nonassociative ring and let $\mathcal{I}(R)$ denote the family of (two-sided) ideals in R .

DEFINITION 2.1. We define a $*$ -operation as a mapping of $\mathcal{I}(R) \times \mathcal{I}(R)$ into the family of additive subgroups of R such that

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