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STOCHASTIC DIFFERENTIAL EQUATIONS FOR THE NON LINEAR FILTERING PROBLEM*

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1. Introduction

The general nonlinear filtering or estimation problem may be described as follows. x_t, $(0 \leq t \leq T)$, called the signal or system process is a stochastic process direct observation is not possible. The data concerning x_t is provided by observation on another process z_t which is related to x_t by the model (2.1). (See Sections 2 and 4 for notation and precise definitions). In general it is assumed that x_t takes values in a complete separable metric space while z_t is an n-dimensional process. The least squares estimate of $f(x_t)$ (where f is a suitable real valued function) based on the observations $(z_\tau, 0 \leq \tau \leq t)$ is given by the conditional expectation $E[f(x_t) | z_\tau, 0 \leq \tau \leq t]$. In the general case this estimate depends non-linearly on the observations and is known as the non linear filter. A "Bayes" formula for the conditional expecation has been given in [9] but is useful in applications only when t is fixed. If the data is coming in continuously and we require an estimate which can be continuously revised to take into account the new data, this formula, while valid, is not practical since the estimate at a future time $t+\Delta$ must be computed using all the past data. The formula computed for time t is of no help in computing the estimate at $t+\Delta$. A practical as well as mathematically more interesting way of doing this is by obtaining a stochastic differential equation for the filter.

This problem has acquired a growing literature in recent years. The papers having a direct bearing on our results are the ones by Kallianpur and Striebel ([10], [11]), Shiryaev [18] and Liptzer and Shiryaev [13]. In [18] x_t is assumed to be a Markovian jump process and in [13] the system and observation processes are components of a diffusion process governed by a stochastic differential equation. The results closest in spirit to the present paper are those in [11] where x_t is a Markov process in R^n and independent of the "noise" process w_t. In that paper the Bayes formula for the filter given in [9] is used in deriving the corresponding stochastic differential equation.

Our paper differs from the above mentioned work in two essential res-

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