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ON THE WHITEHEAD GROUP OF THE DIHEDRAL GROUP OF ORDER 2p

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1. In [5], Lam has shown that the Whitehead group $\mathrm{Wh}(S_3)$ of the symmetric group S_3 on three letters is zero. His method is the interesting one which combines the induction theorem with some concrete computations.

The object of this paper is to show the following results.

Theorem. *If G is the dihedral group of order $2p$, p an odd prime, then the Whitehead group* $\mathrm{Wh}(G)$ *of G is torsion free.*

Hence, from the generalized unit theorem by Bass ([1]), we can easily see that $\mathrm{Wh}(G)$ is a free abelian group of rank $(p-3)/2$. In case $p=3$, G is isomorphic to S_3, so our result includes Lam's as a special case.

Our method is essentially based on his idea. However, using some techniques in algebraic K-theory we have been able to simplify the computations, for example, of the reduced norm.

Let $\mathrm{K}_1(ZG)$ be the Whitehead group of the integral group ring ZG of a finite group G. We shall denote by $\mathrm{Wh}(G)$ the cokernel of the natural homomorphism

$$\pm G \xrightarrow{\subset} GL_1(ZG) \xrightarrow{\subset} GL(ZG) \longrightarrow \mathrm{K}_1(ZG)\,,$$

and call it the "Whitehead group of the group G". For any Z-order A in a finite semi-simple Q-algebra, we shall denote by $\mathrm{SK}_1(A)$ the kernel of the reduced norm of $\mathrm{K}_1(A)$, and for any two-sided ideal $\mathfrak{a}$ of A, $\mathrm{SK}_1(A, \mathfrak{a})$ will denote the inverse image of $\mathrm{SK}_1(A)$ with the natural homomorphism $\mathrm{K}_1(A, \mathfrak{a}) \to \mathrm{K}_1(A)$.

2. The following notations will be fixed throughout this paper.

p=any odd prime
G=the dihedral group generated by the elements s and t under the defining relations $s^p=t^2=1$ and $ts=s^{-1}t$
ζ=a primitive p th root of unity
$L=Q(\zeta)$ the cyclotomic field over the rational number field Q
$L_0=Q(\vartheta)$, $\vartheta=\zeta+\zeta^{-1}$, the maximal real subfield of L
$R=Z[\zeta]$ the integral closure in L over the ring Z of rational integers
$R_0=Z[\vartheta]$ the integral closure in L_0 over Z