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FLOW EQUIVALENCE OF DIFFEOMORPHISMS

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1. Introduction

Two dynamical systems are *equivalent* if there is a diffeomorphism h from one manifold to the other such that h maps every orbit of the dynamical system onto an orbit of the other preserving the natural orientations of orbits. For a diffeomorphism f on X, a dynamical system (M, ϕ) is constructed canonically as follows; $M = R \times X/(t, x) \sim (t+1, f^{-1}(X))$ and the flow ϕ is the one which is induced from the natural flow ψ on $R \times X$. Where, ψ is the 1-parameter group given by $\psi_t(u, x) = (u+t, x)$. This (M, ϕ) is called the *suspension* of f.

Let f and g be diffeomorphisms on X and Y respectively. If the suspensions of f and g are equivalent, the paris (X, f) and (Y, g) will be said to be *flow equivalent*.

By S. Smale ([4], [5]), it is shown that *if f and g are conjugate by a diffeomorphism $X \to Y$, then (X, f) and (Y, g) are flow equivalent.* In [2], the following result is shown. *Suppose that there exists no surjection from the fundamental group of X onto the infinite cyclic group* Z. *Then f and g are conjugate if and only if (X, f) and (Y, g) are flow equivalent.* If there is a surjection $\pi_1(X) \to Z$, there is an example of (X, f) and (Y, g) such that (X, f) and (Y, g) are flow equivalent but f and g are not conjugate. In [2] this example is shown when $X = Y = S^1$.

In §3 of this paper, we will show a sufficient condition on (X, f) and (Y, g) under which they will be flow equivalent, and will show examples of (X, f) and (Y, g) such that they will be flow equivalent but f and g will be not conjugate.

In §5, some results about flow equivalence of diffeomorphisms are mentioned. Our main result in this paper are Corollary (5.4) and Theorem (5.5), which can be simplified as follows.

Theorem A. *Let X, Y be compact connected manifolds which may possibly have boundaries. If (X, f) and (Y, g) are flow equivalent, then there exist regular-coverings $p: W \to X$ and $q: W \to Y$ with the common connected covrineg space W, such that both covering transformation groups are isomorphic to Z or trivial group* 1.

Theorem B. *Let (X, f) and (Y, g) be as in Theorem A. Then there exist coverings p: $W \to X$ and q: $W \to Y$ as in Theorem A such that for some pairs of*