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MIXED PROBLEM FOR THE WAVE EQUATION WITH AN OBLIQUE DERIVATIVE BOUNDARY CONDITION

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1. Introduction

Consider a mixed problem

$$(1.1)\qquad \begin{cases} \square u = \left(\dfrac{\partial^2}{\partial t^2}-\dfrac{\partial^2}{\partial x^2}-\dfrac{\partial^2}{\partial y^2}\right)u(x, y, t) = f(x, y, t) & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{in} \quad \Omega\times(0, T) & \\ Bu = \left(b_1(x, y)\dfrac{\partial}{\partial x}+b_2(x, y)\dfrac{\partial}{\partial y}\right)u(x, y, t) = g(x, y, t) & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on} \quad S\times(0, T) & \\ u(x, y, 0) = u_0(x, y) & \\ \dfrac{\partial u}{\partial t}(x, y, 0) = u_1(x, y) & \end{cases}$$

where $S=\partial\Omega$ is a C^∞ simple and compact curve in R^2 and $b_i(x, y)$ $(i=1, 2)$ are real-valued C^∞-functions defined on S. We assume that $(b_1(x, y), b_2(x, y))$ is not tangential to S, i.e. $b_1n_1+b_2n_2\neq 0$ on S, where $n(x, y)=(n_1(x, y), n_2(x, y))$ is the unit outer normal of S at $(x, y)\in S$. The boundary operator B is called an oblique derivative when

$$(1.2)\qquad b_1(x, y)n_2(x, y)-b_2(x, y)n_1(x, y)\neq 0 \qquad \text{on } S.$$

In this paper we consider the mixed problem (1.1) under the condition (1.2).

In recent years mixed problems for hyperbolic equations have been studied by many authors and the general theory developed (for example S. Agmon [1], T. Balaban [2], H.O. Kreiss [9], R. Sakamoto [10]). Concerning second order equations, the problems with the Dirichlet boundary condition and with the Neumann boundary condition are studied satisfactorily. The author showed the well-posedness in L^2-sense of the problems with a fairly general first order derivative boundary condition in [5]. But the problem (1.1) is not contained, under the condition (1.2), in the results of [1], [2], [5], [7], [9] or [10]. Concerning the problem (1.1), we showed its ill-posedness in L^2-sense when a domain is