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ON THE CONVERGENCE OF SUMS OF INDEPENDENT BANACH SPACE VALUED RANDOM VARIABLES

KIYOSI ITÔ AND MAKIKO NISIO

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1. Introduction

The purpose of this paper is to discuss the convergence of sums of independent random variables with values in a separable real Banach space and to apply it to some problems on the convergence of the sample paths of stochastic processes.

For the real random variables, we have a complete classical theory on the convergence of independent sums due to P. Lévy, A. Khinchin and A. Kolmogorov. It can be extended to finite dimensional random variables without any change. In case the variables are infinite dimensional, there are several points which need special consideration. The difficulties come from the fact that bounded subsets of Banach space are not always conditionally compact.

In Section 2 we will discuss some preliminary facts on Borel sets and probability measures in Banach space. In Section 3 we will extend P. Lévy's theorem. In Section 4 we will supplement P. Lévy's equivalent conditions with some other equivalent conditions, in case the random variables are symmetrically distributed. Here the infinite dimensionality will play an important role. The last section is devoted to applications.

2. Preliminary facts

Throughout this paper, E stands for a separable real Banach space and the topology in E is the norm topology, unless stated otherwise. E^* stands for the dual space of E, $\mathcal{B}$ for all Borel subsets of E and $\mathcal{P}$ for all probability measures on $(E, \mathcal{B})$.

The basic probability measure space is denoted by $(\Omega, \mathcal{F}, P)$ and the generic element of Ω by ω. An E-valued random variable X is a map of Ω into E measurable $(\mathcal{F}, \mathcal{B})$. The probability law μ_X of X is a probability measure in $(E, \mathcal{B})$ defined by

$$\mu_X(B) = P(X \in B), \qquad B \in \mathcal{B}.$$

According to Prohorov [5], every $\mu \in \mathcal{P}$ is tight, i.e.