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ON INDUCED REPRESENTATIONS

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Let G be a locally compact topological group and K a compact subgroup of G. For any irreducible unitary representation σ of K, we denote by $U(\sigma)$ the induced representation generated by σ (see §1). In general, $U(\sigma)$ is not irreducible.

The purpose of this paper is to give a method of extracting the irreducible components of $U(\sigma)$ when G is one of the special types of Lie groups.

1. Let G be a connected non-compact semisimple Lie group with a finite dimensional faithful representation and K a maximal compact subgroup of G. We assume that rank G=rank K. For any given irreducible unitary representation σ of K on a representation space V, we can construct a unitary representation $U(\sigma)$ of G as follows. Let $\mathfrak{H}(\sigma)$ be the set of all "Haar-measurable" V-valued functions f which satisfy the following conditions;

$$f(kx) = \sigma(k)f(x) \qquad (k\in K,\ x\in G)$$

and

$$||f||^2 = \int_G ||f(x)||_V^2\, dx < \infty$$

where $||\ ||_V$ denotes the norm in V.

Then $\mathfrak{H}(\sigma)$ is a Hilbert space if we identify functions which differ only on subsets of G of Haar measure zero. The inner product $(\ ,\ )$ in $\mathfrak{H}(\sigma)$ is given by

$$(f_1, f_2) = \int_G (f_1(x), f_2(x))_V dx \qquad (f_1, f_2\in \mathfrak{H}(\sigma))$$

where $(\ ,\ )_V$ denotes the inner product in V. Finally for any $g\in G$, $U_g(\sigma)$ is defined by

$$(U_g(\sigma)f)(x) = f(xg) \qquad (f\in\mathfrak{H}(\sigma),\ x\in G)\,.$$

Thus we obtained the induced representation $U(\sigma)$ generated by σ (cf. [7] (d)).

Our aim is to find out an irreducible closed subspace of $\mathfrak{H}(\sigma)$.

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