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ON CONGRUENT AXIOMS IN LINEARLY ORDERED SPACES, I

SHIGERU KATAYAMA and HIDETAKA TERASAKA

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1. Introduction

In connection with the axioms of congruence of segments on a straight line given in Hilbert's Grundlagen der Geometrie, we will set up a group of axioms of congruence on a linearly ordered space and study their mutual dependency and independency.

In the following let L be a linearly ordered space, that is a set of points, in which for any pair of distinct points A and B either of the relations $A<B$ and $B<A$ holds, and for any three points A, B and C, if $A<B$ and $B<C$ then $A<C$.

When we write AB, it will be understood that A and B are distinct points of L such that $A<B$. AB will be called a *segment*. We write $AC\equiv AB+BC$ if and only if $A<B<C$.

The axioms we are going to study is the following:

Axiom E (UNIQUE EXISTENCE): $\forall AB\ \forall A'\ \exists_1 B'$: $AB=A'B'$, that is, *for any segment AB and for any point A' there is one and only one point B' such that*

$$AB=A'B'.$$

Axiom R (REFLEXIVITY): $AB=AB$.

Axiom S (SYMMETRICITY): $AB=A'B' \Rightarrow A'B'=AB$.

Axiom T (TRANSITIVITY): $AB=A'B',\ A'B'=A''B'' \Rightarrow AB=A''B''$.

Axiom A (ADDITIVITY):

$$AC\equiv AB+BC,\ A'C'\equiv A'B'+B'C',\ AB=A'B',\ BC=B'C' \Rightarrow AC=A'C'.$$

The following scheme will be used in application:

$$\left.\begin{array}{l} AC\equiv AB+BC\,, \\ A'C'\equiv A'B' \\ \qquad +B'C'\,, \\ AB=A'B'\,, \\ BC=B'C' \end{array}\right\} \overset{(\mathrm{A})}{\Longrightarrow} AC=A'C'\,.$$