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MULTIPLICATIVE STRUCTURES IN MOD q COHOMOLOGY THEORIES II

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This paper is the continuation of part I, Osaka J. Math. 2 (1965), pp. 71-115. §§1-5 are contained in Part I and this part consists of §§6-12. We use all notations and notions defined in Part I.

In §6 we discuss admissible multiplications μ_q in $\tilde{K}(\ ; Z_q)$ and show that they induce multiplications μ_q^* in periodic cohomology $\tilde{K}^*(\ ; Z_q)$; Künneth isomorphism $\tilde{K}^*(X; Z_p) \otimes \tilde{K}^*(Y; Z_p) \cong \tilde{K}^*(X \wedge Y; Z_p)$ holds for any prime p ; an important property of mod q K -theory, Propositions 6.3 and 6.4, is discussed. §7 is devoted to the discussion of commutativity criteria of admissible multiplications (Corollary 7.7, Theorems 7.11 and 7.13); we can establish the existence or non-existence of commutative admissible multiplications in $\tilde{K}(\ ; Z_q)$ for all $q > 1$. §8 is a preparation mainly for §9. The existence of associative admissible multiplications is proved in §9 ($q=2$) and §10 ($q \neq 2$). In case $q=2$, it is guaranteed whenever $\eta^{**}=0$ (which is required even for the existence proof of admissible μ_2 of Theorem 5.9) (Theorem 9.9). In case $q \neq 2$, it is proved only under some conditions (Theorems 10.6 and 10.7). These are sufficient to prove the associativity for every admissible multiplication of $\tilde{K}(\ ; Z_q)$ (Corollary 10.8). In §11 we discuss Bockstein spectral sequences for general cohomologies and multiplicative structures in them. We see many analogous properties as those of ordinary Bockstein spectral sequences. Whenever the existence of admissible μ_p is guaranteed by Theorem 5.9, then some μ_p induces multiplications m_r in E_r -terms of mod p Bockstein spectral sequences for each prime p . It is noticeable that d_r behaves as a derivation to m_r (Theorem 11.10) even though the compatibility of the reduction $\rho_{sp,p}$ with μ_{sp} and μ_p is generally not proved, from which follows Künneth's isomorphism for each term of Bockstein spectral sequences of periodic K^* -cohomology (Theorem 11.11). §12 is an appendix treating some further properties of the maps $\bar{a}: M_q \rightarrow M_r$ of 2.4, not treated there.

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