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ON CERTAIN COHOMOLOGY GROUPS ATTACHED TO HERMITIAN SYMMETRIC SPACES

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This is a continuation of our previous paper [9]. In the paper [9] we have attached to a bounded symmetric domain X and a discontinuous group Γ operating on X two kinds of cohomology groups. The first one is associated with a certain representation of Γ and the second one is associated with a so-called canonical automorphic factor. The first purpose of the present paper is to prove Theorem 7.1 which establishes an isomorphism between these two kinds of cohomology groups of type $(0, q)$. This isomorphism has been proved in [9] in the case $q=\dim_C X$ as a generalization of a theorem of Eichler-Shimura, and thus Theorem 7.1 completes our previous result. Before arriving at this theorem, we shall first summarize and reformulate results in [9] in terms of Lie algebra cohomology and we shall give in §5 a decomposition of the laplacian operators Δ', Δ'' into sums of the "representation parts" Δ'_ρ, Δ''_ρ and the "differential parts" Δ'_D, Δ''_D. These decompositions of the laplacians will be utilized throughout the present paper.

The second purpose of this paper is to prove vanishing theorems for the cohomology groups in question. Our results are stated in the forms of Theorems 8.2, 9.1, 9.2, 12.1 and 12.2. The main idea for establishing these vanishing theorems is to relate our cohomology groups, via the decompositions of the laplacians, with the cohomology groups of the abelian Lie algebras $\mathfrak{n}^\pm$. The representation parts of the laplacians are closely related to the laplacians which appeared in the paper of Kostant [8]. The cohomology theory of the Lie algebras $\mathfrak{n}^\pm$ is the object of Kostant's paper and we shall use his main results in the last three sections. As we shall see, one of the essential points in our proof of the vanishing theorems is in fact the vanishing of the cohomology groups of the Lie algebras $\mathfrak{n}^\pm$. We note also that Theorems 9.1 and 9.2 generalize and improve the results of Calabi-Vesentini [3] and Ise [7]. It should be emphasized here that Theorems 9.1 and 9.2 are deduced, owing to the isomorphsm established by Theorem 7.1 between the two kinds of cohomologies of type $(0, q)$, from the corresponding vanishing