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Osaka Math. J.
14 (1962), 53-70.

ON NILPOTENT-FREE MULTIPLICATIVE SYSTEMS

Dedicated to Professor K. Shoda on his sixtieth birthday

By

KENTARO MURATA

In his study of multiplicative systems, the author [8], [9] has defined the accessible join-generator system, and utilized it for decompositions of elements of multiplicative lattices (m-lattices) and of ideals of multiplicative systems. The concept[0)] of accessible join-generator systems seems to be important to develop the algebraic theory of many sorts of lattices, ideal lattices and other multiplicative systems having no usual finite conditions.

The purpose of the present paper is to study the nilpotent-free multiplicative systems. The fundamental concept in our investigation is some restricted accessible join-generator systems of multiplicative systems.

In §1, we shall consider a complete m-lattice (not necessarily commutative nor associative), and obtain a condition to be nilpotent-free. The subject is largely based on the results of the rings having no nilpotent ideals, which are studied by McCoy [6], Levitzki [5] and Nagata [11]. §2 is concerned with some annihilators of elements in a commutative but not necessarily associative nilpotent-free m-lattice. The results of this section are useful in the next one. §3 treats a group having no solvable normal subgroup. Some properties of such a group which have already studied by the author [10] will be used under a suitable restriction. In §§4 and 5, we shall show the analogous results of §§1 and 2 in the case of an associative but not necessarily commutative multiplicative system. The results in these two sections are applicable to the family of ideals of general ring-systems, and which will be shown in the last section.

Throughout this paper, the symbols $\vee$ and $\wedge$ will denote respectively the set-theoretic union and the intersection. By $\{a\,;\,a$ has property $P\}$ we mean the set of all elements a having property P.

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0) From purely lattice-theoretical stand point of view, R. P. Dilworth and Peter Crawley have recently emphasized the importance of such concept. Cf. [3] and [4].