Hashizume, Yoko
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On the Uniqueness of the Decomposition of a Link

By Yoko HASHIZUME

§ 0. Introduction.

A link of multiplicity n is a collection of n disjoint simple closed oriented polygons in the 3-sphere S^3. Especially a link of multiplicity 1 is a so-called knot. H. Schubert [1] showed that the genus of the product of two knots is equal to the sum of their genera and that every knot is decomposable in a unique way into prime knots. The purpose of this paper is to extend his results to the case of links.

In § 1, using some of the results and methods due to H. Schubert [1], we define the product of links and prove some preliminary theorems.

In § 2, we define decomposition systems for non trivial and non separable links, and prove by the aid of the decomposition system the following

MAIN THEOREM. *Every non trivial and non separable link can be decomposed uniquely into prime links.*

For the links of multiplicity 1, this theorem coincides with H. Schubert's result. But our proof is simpler than his.

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§ 1. A *link of multiplicity* n is a collection of n disjoint simple closed oriented polygons in the 3-sphere S^3 [1]. Two links l and l' are said to be *equivalent* and denoted by $l \approx l'$, if there exists an orientation preserving semilinear mapping S^3 onto itself which maps one of them onto the other. Especially, a link of multiplicity 1 is a so-called knot. Throughout this paper we shall denote by l a link, and by k a knot.

We shall say that l *has* μ *components,* if there are μ disjoint cubes $Q_1, \cdots, Q_\mu$ [2] for l such that $l \frown \dot{Q}_i = \emptyset$, $l \frown Q_i \neq \emptyset$ $(i = 1, \cdots, \mu)$ and there are no $\mu + 1$ disjoint cubes with these properties. A link with multiplicity

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1) In the following S^3 will be taken as the boundary of a 4-simplex in 4-dimensional Euclidean space R^4. To simplify our observation, we chose "infinity" of S^3 as a vertex of this 4-simplex, and call the opposite 3-simplex the base simplex and denote it by Δ^3.

2) Such expressions as cubes, spheres, surfaces, disks, arcs, etc. should be understood all simplicial and mappings should be understood all semilinear S^3 onto itself.